CSIR-UGC (NET) Exam for Award of Junior Research Fellowship and Eligibility for Lecturership shall be a Single Paper Test having Multiple Choice Questions (MCQs). The question paper shall be divided in three parts.

Part A
This part shall carry 20 questions pertaining to General Science, Quantitative Reasoning & Analysis and Research Aptitude. The candidates shall be required to answer any 15 questions. Each question shall be of two marks. The total marks allocated to this section shall be 30 out of 200.

Part 'B'
This part shall contain 40 Multiple Choice Questions (MCQs) generally covering the topics given in the syllabus. A candidate shall be required to answer any 25 questions. Each question shall be of three marks. The total marks allocated to this section shall be 75 out of 200.

Part 'C'
This part shall contain 60 questions that are designed to test a candidate's knowledge of scientific concepts and/or application of the scientific concepts. The questions shall be of analytical nature where a candidate is expected to apply the scientific knowledge to arrive at the solution to the given scientific problem. The questions in this part shall have multiple correct options. Credit in a question shall be given only on identification of ALL the correct options. No credit shall be allowed in a question if any incorrect option is marked as correct answer. No partial credit is allowed. A candidate shall be required to answer any 20 questions. Each question shall be of 4.75 marks. The total marks allocated to this section shall be 95 out of 200.

For Part ‘A’ and ‘B’ there will be Negative marking @25% for each wrong answer. No Negative marking for Part ‘C’.

To enable the candidates to go through the questions, the question paper booklet shall be distributed 15 minutes before the scheduled time of the exam. The Answer sheet shall be distributed at the scheduled time of the exam.

On completion of the exam i.e. at the scheduled closing time of the exam, the candidates shall be allowed to carry the Question Paper Booklet. No candidate is allowed to carry the Question Paper Booklet in case he/she chooses to leave the test before the scheduled closing time.
Joint CSIR-UGC NET for JRF and Eligibility for Lectureship

Notice

It is notified for information of all students that syllabus of Part A of Joint CSIR-UGC Test for Junior Research Fellowship and Eligibility for Lectureship has been revised. The existing syllabus and revised syllabus is as under:

<table>
<thead>
<tr>
<th>Existing Syllabus</th>
<th>Revised Syllabus</th>
</tr>
</thead>
<tbody>
<tr>
<td>General Science, Quantitative Reasoning and Analysis and Research Aptitude.</td>
<td>General Aptitude with emphasis On logical reasoning, graphical analysis, analytical and numerical ability, quantitative comparison, series formation, puzzles etc.</td>
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The number of questions available and to be attempted in this Part A remains the same i.e. there will be 20 questions and the candidates shall be required to answer any 15 questions. Each question will be of two marks.

The model question in the revised syllabus are also available.

****
Analysis: Elementary set theory, finite, countable and uncountable sets, Real number system as a complete ordered field, Archimedean property, supremum, infimum.

Sequences and series, convergence, limsup, liminf.

Bolzano Weierstrass theorem, Heine Borel theorem.

Continuity, uniform continuity, differentiability, mean value theorem.

Sequences and series of functions, uniform convergence.

Riemann sums and Riemann integral, Improper Integrals.

Monotonic functions, types of discontinuity, functions of bounded variation, Lebesgue measure, Lebesgue integral.

Functions of several variables, directional derivative, partial derivative, derivative as a linear transformation, inverse and implicit function theorems.

Metric spaces, compactness, connectedness. Normed linear Spaces. Spaces of continuous functions as examples.

Linear Algebra: Vector spaces, subspaces, linear dependence, basis, dimension, algebra of linear transformations.

Algebra of matrices, rank and determinant of matrices, linear equations.

Eigenvalues and eigenvectors, Cayley-Hamilton theorem.

Matrix representation of linear transformations. Change of basis, canonical forms, diagonal forms, triangular forms, Jordan forms.

Inner product spaces, orthonormal basis.

Quadratic forms, reduction and classification of quadratic forms

Complex Analysis: Algebra of complex numbers, the complex plane, polynomials, power series, transcendental functions such as exponential, trigonometric and hyperbolic functions. Analytic functions, Cauchy-Riemann equations.
Contour integral, Cauchy’s theorem, Cauchy’s integral formula, Liouville’s theorem, Maximum modulus principle, Schwarz lemma, Open mapping theorem.

Taylor series, Laurent series, calculus of residues.

Conformal mappings, Mobius transformations.

**Algebra:** Permutations, combinations, pigeon-hole principle, inclusion-exclusion principle, derangements.

Fundamental theorem of arithmetic, divisibility in \( \mathbb{Z} \), congruences, Chinese Remainder Theorem, Euler’s \( \varphi \)-function, primitive roots.

Groups, subgroups, normal subgroups, quotient groups, homomorphisms, cyclic groups, permutation groups, Cayley’s theorem, class equations, Sylow theorems.

Rings, ideals, prime and maximal ideals, quotient rings, unique factorization domain, principal ideal domain, Euclidean domain.

Polynomial rings and irreducibility criteria.

Fields, finite fields, field extensions, Galois Theory.

**Topology:** basis, dense sets, subspace and product topology, separation axioms, connectedness and compactness.

**UNIT – 3**

**Ordinary Differential Equations (ODEs):**

Existence and uniqueness of solutions of initial value problems for first order ordinary differential equations, singular solutions of first order ODEs, system of first order ODEs.

General theory of homogenous and non-homogeneous linear ODEs, variation of parameters, Sturm-Liouville boundary value problem, Green’s function.

**Partial Differential Equations (PDEs):**

Lagrange and Charpit methods for solving first order PDEs, Cauchy problem for first order PDEs.

Classification of second order PDEs, General solution of higher order PDEs with constant coefficients, Method of separation of variables for Laplace, Heat and Wave equations.

**Numerical Analysis:**

Numerical solutions of algebraic equations, Method of iteration and Newton-Raphson method, Rate of convergence, Solution of systems of linear algebraic equations using Gauss elimination and Gauss-Seidel methods, Finite differences, Lagrange, Hermite and spline interpolation, Numerical differentiation and integration, Numerical solutions of ODEs using Picard, Euler, modified Euler and
Runge-Kutta methods.

**Calculus of Variations:**


**Linear Integral Equations:**

Linear integral equation of the first and second kind of Fredholm and Volterra type, Solutions with separable kernels. Characteristic numbers and eigenfunctions, resolvent kernel.

**Classical Mechanics:**

Generalized coordinates, Lagrange’s equations, Hamilton’s canonical equations, Hamilton’s principle and principle of least action, Two-dimensional motion of rigid bodies, Euler’s dynamical equations for the motion of a rigid body about an axis, theory of small oscillations.

**UNIT – 4**

Descriptive statistics, exploratory data analysis

Sample space, discrete probability, independent events, Bayes theorem. Random variables and distribution functions (univariate and multivariate); expectation and moments. Independent random variables, marginal and conditional distributions. Characteristic functions. Probability inequalities (Tchebyshef, Markov, Jensen). Modes of convergence, weak and strong laws of large numbers, Central Limit theorems (i.i.d. case).


Standard discrete and continuous univariate distributions. sampling distributions, standard errors and asymptotic distributions, distribution of order statistics and range.

Methods of estimation, properties of estimators, confidence intervals. Tests of hypotheses: most powerful and uniformly most powerful tests, likelihood ratio tests. Analysis of discrete data and chi-square test of goodness of fit. Large sample tests.

Simple nonparametric tests for one and two sample problems, rank correlation and test for independence. Elementary Bayesian inference.


Simple random sampling, stratified sampling and systematic sampling. Probability proportional to size sampling. Ratio and regression methods.

Completely randomized designs, randomized block designs and Latin-square designs. Connectedness and orthogonality of block designs, BIBD. 2x factorial experiments: confounding and construction.

Hazard function and failure rates, censoring and life testing, series and parallel systems.

Linear programming problem, simplex methods, duality. Elementary queuing and inventory models. Steady-state solutions of Markovian queuing models: M/M/1, M/M/1 with limited waiting space, M/M/C, M/M/C with limited waiting space, M/G/1.

All students are expected to answer questions from Unit I. Students in mathematics are expected to answer additional question from Unit II and III. Students with in statistics are expected to answer additional question from Unit IV.
MODEL QUESTION PAPER

PART 'A' GENERAL APTITUDE

(logical reasoning, graphical analysis, analytical and numerical ability, quantitative comparisons, series formation, puzzles, etc.)
1. A sphere of radius 4 cm is carved from a homogeneous sphere of radius 8 cm and mass 160 g. The mass of the smaller sphere is
   
   1. 80 g.
   2. 60 g.
   3. 40 g.
   4. 20 g.

2. Two boys A and B are at two diametrically opposite points on a circle. At one instant the two start running on the circle; A anticlockwise with constant speed $v$ and B clockwise with constant speed $2v$. In 2 minutes, they pass each other for the first time. How much later will they pass each other for the second time?
   
   1. 1 minute
   2. 2 minutes
   3. 3 minutes
   4. 4 minutes

3. There are $k$ baskets and $n$ balls. The balls are put into the baskets randomly. If $k < n$,
   
   1. There is no empty basket
   2. There are exactly $(n-k)$ baskets with at least one ball
   3. There is at least one basket with two or more balls
   4. There are $(n-k)$ baskets with exactly two balls

4. An ant is crawling along the x-axis such that the graph of its position on the x-axis versus time is a semi-circle (see figure). The total distance covered in the 4 s is
   
   ![Graph of semi-circle](image)

   1. 4 m
   2. 2 m
   3. $2\pi$ m
   4. $4\pi$ m
5. In a bag containing only blue, red and green marbles, all but 15 are blue, all but 13 are red and all but 12 are green. How many are red?

1. 13
2. 7
3. 25
4. 20

6. Find the missing numbers in the bottom middle circle. (Clue: left halves of the central circles relate to the left circles and the right halves to the right circles)

```
5 6 30 3 3 9
```

```
1 6
```

1. 10, 20
2. 15, 15
3. 21, 2
4. 6, 2

7. Identify the missing letter.

```
B Y
V E
M N
Z ?
```

1. W
2. A
3. X
4. B
8. Identify the figure that comes next in the sequence.

![Figure sequence]

9. A person buys a shirt with marked price Rs.300/- at 20% discount. In order to make a profit of 20% the person should sell the shirt for

1. Rs. 288/-
2. Rs. 300/-
3. Rs. 240/-
4. Rs. 360/-

10. A uniform cylindrical container is half filled with water. The height of the cylinder is twice its diameter. The cylinder is gradually tilted until the water touches the brim. At this instant, the container is inclined at

1. 30° to vertical
2. 45° to vertical
3. 60° to vertical
4. 75° to vertical

11. A car is moving along a straight road. The graph below shows how the speed varies with time.

![Graph]

Which of the following graphs represents the distance covered by the car with time?
12. In triangle ABC, shown in the figure, AB is perpendicular to BC. Further, BD is perpendicular to AC. If AD = 9 cm and DC = 4 cm, the length BD is

\[
\text{BD} = \sqrt{AD^2 + DC^2} = \sqrt{9^2 + 4^2} = \sqrt{81 + 16} = \sqrt{97}
\]

1. 6 cm
2. 6.5 cm
3. \(\frac{36}{13}\) cm
4. \(\frac{13}{36}\) cm

13. A box of sticks of equal lengths is provided. The minimum number sticks needed to build a frame to enclose a 3 dimensional volume is

1. 6
2. 12
3. 3
4. 8
14. 5 kg of adulterated rice has 2% stones in it and the rest is rice. Half of the stone content was removed. Now the percentage of stone content in it is

1. 0.99%
2. 1%
3. 1.1%
4. 1.01%

15. A coin is tossed six times. The probability that heads will occur at least once is

1. \( \frac{63}{64} \)
2. \( \frac{1}{3} \)
3. \( \frac{1}{64} \)
4. \( \frac{3}{2} \)

16. Two parameters TC and TF are related as shown in the table. Find the value of TF corresponding to a TC of 75.

<table>
<thead>
<tr>
<th>TC</th>
<th>0</th>
<th>25</th>
<th>50</th>
<th>75</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>TF</td>
<td>32</td>
<td>77</td>
<td>122</td>
<td>?</td>
<td>212</td>
</tr>
</tbody>
</table>

1. 167
2. 162
3. 150
4. 200

17. Several identical cubes are arranged in a close-packed single layer. If the area of the layer is \( A \) and the volume of the layer is \( V \), then the number of cubes in the collection is

1. \( V/A \)
2. \( A^3/V^2 \)
3. \( V^2/A^3 \)
4. \( A/V \).
18. Circle PQR is inscribed in a quadrilateral ABCD. The circle touches side AD at point S. AP = 8 cm, QC = 3 cm and DC = 6 cm. The length of side AD is

\[ 1. \quad 9 \text{ cm} \]
\[ 2. \quad 10 \text{ cm} \]
\[ 3. \quad 11 \text{ cm} \]
\[ 4. \quad 12 \text{ cm} \]

19. The distribution of wages in a population is shown below for 2 years.

The average wage(s)

1. in 1999 is greater than that in 2000.
2. in 1999 is less than that in 2000.
3. in the two years are equal, but the variances are unequal.
4. in the two years are unequal, but the variances are equal.
The graph below shows crime rates and convictions for 5 years in a certain society.

Which of the following is correct?

1. The cause of convictions being on the rise is better law enforcement.
2. Falling crime rates have slowly reduced the conviction rates also.
3. There are fewer convictions because crime rate has fallen.
4. The graph must be wrong.
1. Profit of a firm grows at a rate of 15% per year for the first three consecutive years. For the next three years, the profit level remains stagnant. From the 6th year till the 9th year, it again grows at a rate of 15% per year. Which of the following graphs depicts these facts?

2. A pond is deepest at its centre and becomes shallow uniformly towards the edge. If the depth of water at the centre in May is half its value in August, the water contained in the pond

   (1) in May is greater than half that in August
   (2) in August is equal to twice that in May
   (3) in May is less than half that in August
   (4) in August is less than twice that in May
3. The series representing the sum of the areas of the shaded equilateral triangles in the figure below is

1. \[3+2+1+1\]
2. \[\frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \frac{1}{3^4} \ldots\]
3. \[\frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3} + \frac{1}{4^4} \ldots\]
4. \[\frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} \ldots\]

4. Which of the following vitamins will not be synthesized in a person confined to a dark cell for a long time?

1. A
2. B
3. C
4. D

5. Flowering is some plants is strongly influenced by the photo period. A farmer was growing two species of plants, A and B near a sea coast where a light house was located. He observed that species A flowered profusely while species B did not. Which of the following is correct?

1. Species A requires long duration of day while species B needs a shorter day
2. Species B requires longer duration of day while species A needs a shorter duration
3. Both species require short duration of day
4. Both species require long duration of day
6. Pneumatophores are modified roots in some plants like *Rhizophora* growing in swampy areas that come out of the ground and grow vertically upwards. The main function of such roots is to
   (1) help obtain oxygen for respiration
   (2) provide support
   (3) adsorb and conduct water and minerals
   (4) store food

7. A cube of side 1 cm is painted by putting a lacquer of thickness \( \delta \), negligible compared to the side of the cube. The volume of the painted cube is approximately

   (1) \( 1 + \delta \) cm\(^3\)
   (2) \( 1 + \delta^3 \) cm\(^3\)
   (3) \( 1 + 3\delta \) cm\(^3\)
   (4) \( 1 + 3\delta^3 \) cm\(^3\)

8. A candle is burning inside a sealed glass jar. The pressure and temperature of the air within the jar are plotted as a function of time. Which of the following graphs represents this process correctly?

9. The result of taking 1’s complement of the sum of the binary numbers 110 and 101 will be

   (1) 1011
   (2) 0011
   (3) 0100
   (4) 0110

10. Which of the following straight lines passes through the point (1,1)?

    (1) \( y = 2x + 3 \)
(2) \(2y = x - 6\)
(3) \(x = 1\)
(4) \(x = y + 1\)

11. Which of the following 1 molar (aqueous) solution has the highest number density of ions?

(1) Glucose
(2) \(\text{CaCl}_2\)
(3) \(\text{NaNO}_3\)
(4) \(\text{KCl}\)

12. How many two-digit even numbers can be composed from nine digits 1, 2, 3 … 9?

(1) 50
(2) 81
(3) 45
(4) 36

13. Complete combustion of cyclohexane (\(\text{C}_6\text{H}_{12}\)) is represented by the equation

\[
\text{C}_6\text{H}_{12} + x \text{O}_2 \rightarrow y \text{CO}_2 + z \text{H}_2\text{O}
\]

The values of \(x\), \(y\) and \(z\), respectively, are

(1) 9, 6, 6
(2) 10, 6, 4
(3) 6, 12, 10
(4) 4, 8, 12

14. How many distinct trichlorobenzenes (\(\text{C}_6\text{H}_3\text{Cl}_3\)) should exist, given that benzene (\(\text{C}_6\text{H}_6\)) has a regular hexagonal geometry?

(1) 6
(2) 1
(3) 2
(4) 3

15. Mercury is closer to the Sun than Venus. Yet Venus is hotter because it has

(1) a dominant \(\text{CO}_2\) atmosphere
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(3) sulphuric acid clouds
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17. A container holding normal air (1 bar pressure, room temperature) is being evacuated. The normal composition of air is approximately 78% N\textsubscript{2}, 21% O\textsubscript{2}, 0.9% Ar and traces of CO\textsubscript{2} (0.04%) and water vapour (0.02%). After the pressure in the container falls to about 10\textsuperscript{-3} mbar, the relative fractions of the components will be

1. N\textsubscript{2} and O\textsubscript{2} approximately equal and greater than H\textsubscript{2}O  
2. N\textsubscript{2}, O\textsubscript{2}, Ar approximately equal and greater than H\textsubscript{2}O  
3. N\textsubscript{2}, O\textsubscript{2}, Ar in the original proportion, but N\textsubscript{2} less than H\textsubscript{2}O  
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18. An endoscope is a device for observing internal organs, using a combination of a lamp and an optical fibre. The image seen is due to

(1) light reflected by the organ and transmitted by internal reflection through the fibre  
(2) light refracted by the organ and transmitted by refraction through the fibre.  
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(1) 24  
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(3) 36  
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20. A typical enzyme catalyzed reaction is shown below

What do you think the component $x$ might be?

1. Substrate concentration or temperature
2. Substrate concentration or enzyme concentration
3. Substrate concentration or pH
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1. Profit of a firm grows at a rate of 15% per year for the first three consecutive years. For the next three years, the profit level remains stagnant. From the 6th year till the 9th year, it again grows at a rate of 15% per year. Which of the following graphs depicts these facts?

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The values of $x$, $y$ and $z$, respectively, are

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3. Substrate concentration or pH
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MATHEMATICAL SCIENCES

This Test Booklet will contain 120 (20 Part ‘A’+40 Part ‘B’+60 Part ‘C’) Multiple Choice Questions (MCQs) Both in Hindi and English. Candidates are required to answer 15 in part ‘A’, 25 in Part ‘B’ and 20 questions in Part ‘C’ respectively (No. of questions to attempt may vary from exam to exam). In case any candidate answers more than 15, 25 and 20 questions in Parts A, B and C respectively only first 15, 25 and 20 questions in Parts A, B and C respectively will be evaluated. Each questions in Parts ‘A’ carries two marks, Part ‘B’ three marks and Part ‘C’ 4.75 marks respectively. There will be negative marking @0.5 marks in Part ‘A’ and 0.75 in part ‘B’ for each wrong answers. Below each question in Part ‘A’ and Part ‘B’, four alternatives or responses are given. Only one of these alternatives is the ‘CORRECT’ answer to the question. Part ‘C’ shall have one or more correct options. Credit in a question shall be given only on identification of ALL the correct options in Part ‘C’. No credit shall be allowed in a question if any incorrect option is marked as correct answer. No partial credit is allowed.

MODEL QUESTION PAPER

PART A

May be viewed under heading “General Science”

PART B

21. The sequence \( a_n = \frac{1}{n^2} + \frac{1}{(n+1)^2} + \ldots + \frac{1}{(2n)^2} \)

1. converges to 0
2. converges to 1/2
3. converges to 1/4
4. does not converge.

22. Let \( x_n = n^{1/n} \) and \( y_n = (n!)^{1/n} \), \( n \geq 1 \) be two sequences of real numbers. Then

1. \((x_n)\) converges, but \((y_n)\) does not converge
2. \((y_n)\) converges, but \((x_n)\) does not converge
3  both \((x_n)\) and \((y_n)\) converge
4  Neither \((x_n)\) nor \((y_n)\) converges

23. The set \(\{ x \in \mathbb{R} : x \sin x \leq 1, x \cos x \leq 1 \} \subset \mathbb{R}\) is
1  a bounded closed set
2  a bounded open set
3  an unbounded closed set.
4  an unbounded open set.

24. Let \(f:[0,1] \rightarrow \mathbb{R}\) be continuous such that \(f(t) \geq 0\) for all \(t\) in \([0,1]\). Define
\[
g(x) = \int_0^x f(t) \, dt\]
then
1  \(g\) is monotone and bounded
2  \(g\) is monotone, but not bounded
3  \(g\) is bounded, but not monotone
4  \(g\) is neither monotone nor bounded

25. Let \(f\) be a continuous function on \([0,1]\) with \(f(0)=1\). Let \(G(a) = \frac{1}{a} \int_0^a f(x) \, dx\)

1  \(\lim_{a \to 0^+} G(a) = \frac{1}{2}\)
2  \(\lim_{a \to 0^+} G(a) = 1\)
3  \(\lim_{a \to 0^+} G(a) = 0\)
4  The limit \(\lim_{a \to 0^+} G(a)\) does not exist

26. Let \(\alpha_n = \sin \left( \frac{1}{n^2} \right), n = 1, 2, \ldots\) Then

1  \(\sum_{n=1}^{\infty} \alpha_n\) converges
2  \(\limsup_{n \to \infty} \alpha_n \neq \liminf_{n \to \infty} \alpha_n\)
3  \(\lim_{n \to \infty} \alpha_n = 1\)
4  \(\sum_{n=1}^{\infty} \alpha_n\) diverges
27. If, for $x \in \mathbb{R}$, $\phi(x)$ denotes the integer closest to $x$ (if there are two such integers take the larger one), then $\int_{10}^{12} \phi(x) \, dx$ equals

\[
\begin{array}{c}
1 \quad 22 \\
2 \quad 11 \\
3 \quad 20 \\
4 \quad 12
\end{array}
\]

28. Let $P$ be a polynomial of degree $k > 0$ with a non-zero constant term. Let $f_n(x) = P\left(\frac{x}{n}\right) \quad \forall x \in (0, \infty)$

\[
\begin{array}{c}
1 \quad \lim_{n \to \infty} f_n(x) = \infty \quad \forall x \in (0, \infty) \\
2 \quad \exists x \in (0, \infty) \text{ such that } \lim_{n \to \infty} f'_n(x) > P(0) \\
3 \quad \lim_{n \to \infty} f'_n(x) = 0 \quad \forall x \in (0, \infty) \\
4 \quad \lim_{n \to \infty} f'_n(x) = P(0) \quad \forall x \in (0, \infty)
\end{array}
\]

29. Let $C[0, 1]$ denote the space of all continuous functions with supremum norm.

Then, $K = \left\{ f \in \hat{I} \mid 0,1 : \lim_{n \to \infty} \int_{\mathbb{R}^3} f \, d\hat{\bigwedge} = 0 \right\}$ is a vector space but not closed in $C[0,1]$.

2. closed but does not form a vector space.
3. a closed vector space but not an algebra.
4. a closed algebra.

30. Let $u, v, w$ be three points in $\mathbb{R}^3$ not lying in any plane containing the origin.

Then

\[
\begin{array}{c}
1 \quad \alpha_1 u + \alpha_2 v + \alpha_3 w = 0 \Rightarrow \alpha_1 = \alpha_2 = \alpha_3 = 0 \\
2 \quad u, v, w \text{ are mutually orthogonal} \\
3 \quad \text{one of } u, v, w \text{ has to be zero} \\
4 \quad u, v, w \text{ cannot be pairwise orthogonal}
\end{array}
\]
31. Let \( x, y \) be linearly independent vectors in \( \mathbb{R}^2 \) suppose \( T: \mathbb{R}^2 \to \mathbb{R}^2 \) is a linear transformation such that \( Ty = \alpha x \) and \( Tx = 0 \) Then with respect to some basis in \( \mathbb{R}^2 \), \( T \) is of the form

1. \[
\begin{pmatrix}
a & 0 \\
0 & a
\end{pmatrix}, \quad a > 0
\]

2. \[
\begin{pmatrix}
a & 0 \\
0 & b
\end{pmatrix}, \quad a, b > 0; \quad a \neq b
\]

3. \[
\begin{pmatrix}
0 & 1 \\
0 & 0
\end{pmatrix}
\]

4. \[
\begin{pmatrix}
0 & 0 \\
0 & 0
\end{pmatrix}
\]

32. Suppose \( A \) is an \( n \times n \) real symmetric matrix with eigenvalues \( \lambda_1, \lambda_2, \ldots, \lambda_n \) then

1. \[
\prod_{i=1}^{n} \lambda_i < \det(A)
\]

2. \[
\prod_{i=1}^{n} \lambda_i > \det(A)
\]

3. \[
\prod_{i=1}^{n} \lambda_i = \det(A)
\]

4. \( \text{if } \det(A) = 1 \text{ then } \lambda_j = 1 \text{ for } j = 1, \ldots, n. \)

33. Let \( f \) be analytic on \( D = \{ z \in \mathbb{C} : |z| < 1 \} \) and \( f(0) = 0 \).

Define

\[
g(z) = \begin{cases} 
\frac{f(z)}{z}; & z \neq 0 \\
 f'(0); & z = 0 
\end{cases}
\]

Then

1. \( g \) is discontinuous at \( z = 0 \) for all \( f \)

2. \( g \) is continuous, but not analytic at \( z = 0 \) for all \( f \)

3. \( g \) is analytic at \( z = 0 \) for all \( f \)

4. \( g \) is analytic at \( z = 0 \) only if \( f'(0) = 0 \)
34. Let $\Omega \subseteq \mathbb{C}$ be a domain and let $f(z)$ be an analytic function on $\Omega$ such that

$$|f(z)| = |\sin z| \text{ for all } z \in \Omega$$

then

1. $f(z) = \sin z$ for all $z \in \Omega$
2. $f(z) = \sin(\bar{z})$ for all $z \in \Omega$.
3. there is a constant $c \in \mathbb{C}$ with $|c| = 1$ such that $f(z) = c \sin z$
   for all $z \in \Omega$
4. such a function $f(z)$ does not exist

35. The radius of convergence of the power series

$$\sum_{n=0}^{\infty} (4n^4 - n^3 + 3) z^n$$

is

1. 0
2. 1
3. 5
4. $\infty$

36. Let $\mathbb{F}$ be a finite field such that for every $a \in \mathbb{F}$ the equation $x^2 = a$ has a solution in $\mathbb{F}$. Then

1. the characteristic of $\mathbb{F}$ must be 2
2. $\mathbb{F}$ must have a square number of elements
3. the order of $\mathbb{F}$ is a power of 3
4. $\mathbb{F}$ must be a field with prime number of elements

37. Let $\mathbb{F}$ be a field with $5^{12}$ elements. What is the total number of proper subfields of $\mathbb{F}$?

1. 3
2. 6
3. 8
4. 5
38. Let $K$ be an extension of the field $\mathbb{Q}$ of rational numbers
   1. If $K$ is a finite extension then it is an algebraic extension
   2. If $K$ is an algebraic extension then it must be a finite extension
   3. If $K$ is an algebraic extension then it must be an infinite extension
   4. If $K$ is a finite extension then it need not be an algebraic extension

39. Consider the group $S_9$ of all the permutations on a set with 9 elements. What is the largest order of a permutation in $S_9$?
   1. 21
   2. 20
   3. 30
   4. 14

40. Suppose $V$ is a real vector space of dimension 3. Then the number of pairs of linearly independent vectors in $V$ is
   1. one
   2. infinity
   3. $e^3$
   4. 3

41. Consider the differential equation
   \[ \frac{dy}{dx} = y^2, \quad (x, y) \in \mathbb{R} \times \mathbb{R}. \]
   Then,
   1. all solutions of the differential equation are defined on $(-\infty, \infty)$.
   2. no solution of the differential equation is defined on $(-\infty, \infty)$.
   3. the solution of the differential equation satisfying the initial condition $y(x_0) = y_0, \ y_0 > 0$, is defined on $\left(-\infty, x_0 + \frac{1}{y_0}\right)$.
   4. the solution of the differential equation satisfying the initial condition $y(x_0) = y_0, \ y_0 > 0$, is defined on $\left(\infty, \frac{1}{y_0}, \frac{\partial}{\partial y}\right)$.

42. The second order partial differential equation
   \[ \left(1 - \sqrt{xy}\right) \frac{\partial^2 u}{\partial x^2} + 2 \frac{\partial^2 u}{\partial x \partial y} + \left(1 + \sqrt{xy}\right) \frac{\partial^2 u}{\partial y^2} = 0 \]
   is
   1. hyperbolic in the second and the fourth quadrants
   2. elliptic in the first and the third quadrants
3. hyperbolic in the second and elliptic in the fourth quadrant
4. hyperbolic in the first and the third quadrants

43. A general solution of the equation $\frac{\partial u(x, y)}{\partial x} + u(x, y) = e^{-x}$ is
   1. $u(x, y) = e^{-x}f(y)$
   2. $u(x, y) = e^{-x}f(y) + xe^x$
   3. $u(x, y) = e^{-x}f(y) + xe^x$
   4. $u(x, y) = e^{-x}f(y) + xe^{-x}$

44. Consider the application of Trapezoidal and Simpson’s rules to the following integral
   \[ \int_{0}^{a} (2x^3 - 3x^2 + 5x + 1) dx \]
   1. Both Trapezoidal and Simpson’s rules will give results with same accuracy.
   2. The Simpson’s rule will give more accuracy than the Trapezoidal rule but less accurate than the exact result.
   3. The Simpson’s rule will give the exact result.
   4. Both Trapezoidal rule and Simpson’s rule will give the exact results.

45. The integral equation
   \[ g(x)y(x) = f(x) + \lambda \int_{a}^{b} k(x,t)y(t) dt \]
   with $f(x)$, $g(x)$ and $k(x,t)$ as known functions, $\alpha$ and $\beta$ as known constants, and $\lambda$ as a known parameter, is a
   1. linear integral equation of Volterra type
   2. linear integral equation of Fredholm type
   3. nonlinear integral equation of Volterra type
   4. nonlinear integral equation of Fredholm type

46. Let $y(x) = f(x) + \lambda \int_{a}^{b} k(x,t)y(t) dt$, where $f(x)$ and $k(x,t)$ are known functions, $a$ and $b$ are known constants and $\lambda$ is a known parameter. If $\lambda$ be the eigenvalues of the corresponding homogeneous equation, then the above integral equation has in general,
   1. many solutions for $\lambda \neq \lambda_i$
   2. no solution for $\lambda = \lambda_i$
   3. a unique solution for $\lambda = \lambda_i$
   4. either many solutions or no solution at all for $\lambda = \lambda_i$, depending on the form of $f(x)$

47. The equation of motion of a particle in the $x$-$z$ plane is given by
   \[ \frac{d\ddot{y}}{dt} = -\dot{\dot{y}} - \dot{k} \]
with \( \vec{v} = \alpha \hat{k} \), where \( \alpha = \alpha(t) \) and \( \hat{k} \) is the unit vector along the z-direction. If initially (i.e., \( t = 0 \)) \( \alpha = 1 \), then the magnitude of velocity at \( t = 1 \) is

1. \( \frac{2}{e} \)
2. \( \frac{2 + e}{3} \)
3. \( \frac{(e - 2)}{e} \)
4. 1

48. Consider the functional

\[
F(u,v) = \int_0^{\pi/2} \left[ \left( \frac{du}{dx} \right)^2 + \left( \frac{dv}{dx} \right)^2 + 2u(x)v(x) \right] \, dx
\]

with

\[
u(0) = 1, v(0) = -1 \text{ and } u\left( \frac{\pi}{2} \right) = 0, v\left( \frac{\pi}{2} \right) = 0.
\]

Then, the extremals satisfy

1. \( u(\pi) = 1, v(\pi) = -1 \)
2. \( u(\pi) + v(\pi) = 0, u(\pi) - v(\pi) = 2 \)
3. \( u(p) = -1, v(p) = 1 \)
4. \( u(\pi) + v(\pi) = -2, u(\pi) - v(\pi) = 0 \)

49. The pairs of observations on two random variables X and Y are

\[
X : 2 5 7 11 13 19
Y : 0 15 25 45 55 85
\]

Then the correlation coefficient between X and Y is

1. 0
2. 1/5
3. 1/2
4. 1

50. Let \( X_1, X_2, X_3 \) be independent random variables with \( P(X_i = +1) = P(X_i = -1) = 1/2 \). Let \( Y_1 = X_2X_3, Y_2 = X_1X_3 \) and \( Y_3 = X_1X_2 \).

Then which of the following is NOT true?

1. \( Y_i \) and \( X_i \) have same distribution for \( i = 1, 2, 3 \)
2. \( (Y_1, Y_2, Y_3) \) are mutually independent
3. \( X_1 \) and \( (Y_2, Y_3) \) are independent
4. \( (X_1, X_2) \) and \( (Y_1, Y_2) \) have the same distribution
51. Let $X$ be an exponential random variable with parameter $\lambda$. Let $Y = [X]$ where $[x]$ denotes the largest integer smaller than $x$. Then

1. $Y$ has a Geometric distribution with parameter $\lambda$.
2. $Y$ has a Geometric distribution with parameter $1 - e^{-\lambda}$.
3. $Y$ has a Poisson distribution with parameter $\lambda$.
4. $Y$ has mean $[1/\lambda]$.

52. Consider a finite state space Markov chain with transition probability matrix $P=([p_{ij}])$. Suppose $p_{ii} = 0$ for all states $i$. Then the Markov chain is

1. always irreducible with period 1.
2. may be reducible and may have period $> 1$.
3. may be reducible but period is always 1.
4. always irreducible but may have period $> 1$.

53. Let $X_1, X_2, \ldots, X_n$ be i.i.d. Normal random variables with mean 1 and variance 1. and let $Z_n = (X_1^2 + X_2 + \ldots + X_n)/n$ Then

1. $Z_n$ converges in probability to 1
2. $Z_n$ converges in probability to 2
3. $Z_n$ converges in distribution to standard normal distribution
4. $Z_n$ converges in probability to Chi-square distribution.

54. Let $X_1, X_2, \ldots, X_n$ be a random sample of size $n \geq 4$ from uniform $(0,\theta)$ distribution. Which of the following is NOT an ancillary statistic?

1. $\frac{X_{(n)}}{X_{(1)}}$
2. $\frac{X_n}{X_1}$
3. $\frac{X_4 - X_1}{X_3 - X_2}$
4. $X_{(n)} - X_{(1)}$
55. Suppose $X_1, X_2, \ldots, X_n$ are i.i.d, Uniform $(0, q)$, $\theta \in \{1, 2, \ldots\}$. Then the MLE of $\theta$ is

1. $X_{(n)}$
2. $\bar{X}$
3. $[X_{(n)}]$ where $[a]$ is the integer part of $a$.
4. $[X_{(n)}+1]$ where $[a]$ is the integer part of $a$.

56. Let $X_1, X_2, \ldots, X_n$ be independent and identically distributed random variables with common continuous distribution function $F(x)$. Let $R_i = \text{Rank}(X_i)$, $i = 1, 2, \ldots, n$. Then $P\left( | R_n - R_1 | \geq n-1 \right)$ is

1. $0$
2. $\frac{1}{n(n-1)}$
3. $\frac{2}{n(n-1)}$
4. $\frac{1}{n}$

57. A simple random sample of size $n$ is drawn without replacement from a population of size $N > n$. If $\pi_i$ ($i=1,2,\ldots,N$) and $p_{ij}$ ($i \neq j, i, j = 1, 2, \ldots N$) denote respectively, the first and second order inclusion probabilities, then which of the following statements is NOT true?

1. $\sum_{i=1}^{N} \pi_i = n$
2. $\sum_{j \neq i}^{N} p_{ij} = (n - 1)p_i$
3. $p_{ij} \leq p_{ij}$ for each pair $(i, j)$
4. $p_{ij} < p_{ij}$ for each pair $(i, j)$.

58. Consider a balanced incomplete block design with usual parameters $v, b, r, k$ ($\geq 2$), $k$. Let $t_i$ be the effect of the $i^{th}$ treatment ($i = 1, 2, \ldots, v$) and $\sigma^2$ denote the variance of an observation. Then the variance of the best linear
unbiased estimator of \( \sum_{i=1}^{\nu} p_{i} l_{i} \) where \( \sum_{i=1}^{\nu} p_{i} = 0 \) and \( \sum_{i=1}^{\nu} p_{i}^2 = 1 \), under the intra-block model, is

1. \( \left( \frac{\lambda v}{k} \right) \sigma^2 \)
2. \( 2\sigma^2 / r \)
3. \( \left( \frac{k}{\lambda v} \right) \sigma^2 \)
4. \( \left( \frac{2k}{\lambda v} \right) \sigma^2 \)

59. An aircraft has four engines – two on the left side and two on the right side. The aircraft functions only if at least one engine on each side functions. If the failures of engines are independent, and the probability of any engine failing in equal to \( p \), then the reliability of the aircraft is equal to

1. \( p^2(1 - p^2) \)
2. \( 4C_2 p^3(1 - p)^2 \)
3. \( (1 - p^2)^2 \)
4. \( 1 - (1 - p^2)^2 \)

60. A company maintains EOQ model for one of its critical components. The setup cost is \( k \), unit production cost is \( c \), demand is \( a \) units per unit time, and \( h \) is the cost of holding one unit per unit time. In view of the criticality of the component the company maintains a safety stock of \( s \) units at all times. The economic order quantity for this problem is given by.

1. \( \sqrt{\frac{2ak}{h}} + s \)
2. \( s + \sqrt{\frac{2ak}{h}} \)
3. \( \sqrt{\frac{2ak}{h}} \)
4. \( \sqrt{\frac{2ak + s}{h}} \)
PART C

61. Suppose \( \{a_n\}, \{b_n\} \) are convergent sequences of real numbers such that \( a_n > 0 \) and \( b_n > 0 \) for all \( n \). Suppose \( \lim_{n \to \infty} a_n = a \) and \( \lim_{n \to \infty} b_n = b \). Let \( c_n = a_n/b_n \). Then

1. \( \{c_n\} \) converges if \( b > 0 \)
2. \( \{c_n\} \) converges only if \( a = 0 \)
3. \( \{c_n\} \) converges only if \( b > 0 \)
4. \( \limsup_{n \to \infty} c_n = \infty \) if \( b = 0 \).

62. Consider the power series \( \sum_{n=0}^{\infty} a_n x^n \)

where \( a_0 = 0 \) and \( a_n = \sin(n!) / n! \) for \( n \geq 1 \). Let \( R \) be the radius of convergence of the power series. Then

1. \( R \geq 1 \)
2. \( R \geq 2\pi \)
3. \( R \leq 4\pi \)
4. \( R \geq \pi \).

63. Suppose \( f \) is an increasing real-valued function on \([0, \infty)\) with \( f(x) > 0 \ \forall x \) and let

\[ g(x) = \frac{1}{x} \int_0^x f(u) \, du; \quad 0 < x < \infty. \]

Then which of the following are true:

1. \( g(x) \leq f(x) \) for all \( x \in (0, \infty) \)
2. \( xg(x) \leq f(x) \) for all \( x \in (0, \infty) \)
3. \( xg(x) \geq f(0) \) for all \( x \in (0, \infty) \)
4. \( yg(y) - xg(x) \leq (y-x)f(y) \) for all \( x < y \).

64. Let \( f: [0, 1] \to \mathbb{R} \) be defined by

\[ f(x) = \begin{cases} x \cos (\pi / (2x)) & \text{if } x \neq 0, \\ 0 & \text{if } x = 0. \end{cases} \]

Then

1. \( f \) is continuous on \([0, 1]\)
2. \( f \) is of bounded variation on \([0, 1]\)
3. \( f \) is differentiable on the open interval \((0, 1)\) and its derivative \( f' \) is bounded on \((0, 1)\)
4. \( f \) is Riemann integrable on \([0, 1]\).

65. For any positive integer \( n \), let \( f_n: [0, 1] \to \mathbb{R} \) be defined by
\[ f_n(x) = \frac{x}{nx+1} \text{ for } x \in [0,1]. \]

Then
1. the sequence \( \{f_n\} \) converges uniformly on \([0, 1]\)
2. the sequence \( \{f'_n\} \) of derivatives of \( \{f_n\} \) converges uniformly on \([0, 1]\)
3. the sequence \( \left\{ \int_0^1 f_n(x)\,dx \right\} \) is convergent
4. the sequence \( \left\{ \int_0^1 f'_n(x)\,dx \right\} \) is convergent.

66. Let \( f: [0, \infty) \to \mathbb{R} \) and \( g : [0, \infty) \to \mathbb{R} \) be continuous functions satisfying

\[ \int_0^{f(x)} t^2 \, dt = x^3 (1 + x)^2 \text{ and } \int_0^{x^2} g(t) \, dt = x \text{ for all } x \in [0, \infty). \]

Then \( f(2) + g(2) \) is equal to
1. 0
2. 5
3. 6
4. 11.

67. Consider \( f: \mathbb{R}^2 \to \mathbb{R} \) defined by \( f(0,0) = 0 \) and

\[ f(x, y) = \frac{x^2 y}{x^4 + y^2} \text{ for } (x, y) \neq (0,0). \]

Then which of the following statements is correct?
1. Both the partial derivatives of \( f \) at \((0,0)\) exist
2. The directional derivative \( D_u f(0,0) \) of \( f \) at \((0,0)\) exists for every unit vector \( u \)
3. \( f \) is continuous at \((0,0)\)
4. \( f \) is differentiable at \((0,0)\).

68. Let \( f: \mathbb{R}^2 \to \mathbb{R} \) and \( g: \mathbb{R}^2 \to \mathbb{R} \) be defined by

\[ f(x, y) = |x| + |y| \text{ and } g(x, y) = |xy|. \]

Then
1. \( f \) is differentiable at \((0,0)\), but \( g \) is not differentiable at \((0,0)\)
2. \( g \) is differentiable at \((0,0)\), but \( f \) is not differentiable at \((0,0)\)
3. Both \( f \) and \( g \) are differentiable at \((0,0)\)
4. Both \( f \) and \( g \) are continuous at \((0,0)\).

69. Decide for which of the functions \( F: \mathbb{R}^3 \to \mathbb{R}^3 \) given below, there exists a function \( f : \mathbb{R}^3 \to \mathbb{R} \) such that \( (\nabla f)(x) = F(x) \).
1. \((4xyz - z^2 - 3y^2, 2x^2z - 6xy + 1, 2x^2y - 2xz - 2)\)
2. \((x, xy, xyz)\)
3. \((1,1,1)\)
4. \((xyz, yz, z)\).

70. Let \(f: \mathbb{R}^n \rightarrow \mathbb{R}\) be the function defined by the rule \(f(x) = x \cdot b\), where \(b \in \mathbb{R}^n\) and \(x \cdot b\) denotes the usual inner product. Then

1. \([f'(x)](b) = b \cdot b\)
2. \([f'(x)](x) = \frac{x \cdot x}{2}, x \in \mathbb{R}^n\)
3. \([f'(0)](e_1) = b \cdot e_1\), where \(e_1 = (1, 0, \ldots, 0) \in \mathbb{R}^n\).
4. \([f'(e_1)](e_j) = 0, j \neq 1\), where \(e_j = (0, \ldots, 1, \ldots, 0)\) with 1 in the \(j\)th slot.

71. Consider the subsets \(A\) and \(B\) of \(\mathbb{R}^2\) defined by

\[
A = \left\{ \left( x, x \sin \frac{1}{x} \right); x \in (0,1] \right\} \quad \text{and} \quad B = A \cup \{ (0,0) \}.
\]

Then
1. \(A\) is compact
2. \(A\) is connected
3. \(B\) is compact
4. \(B\) is connected.

72. Let \(f: \mathbb{R} \rightarrow \mathbb{R}\) be a continuous function. Which of the following is always true?

1. \(f^{-1}(U)\) is open for all open sets \(U \subseteq \mathbb{R}\)
2. \(f^{-1}(C)\) is closed for all closed sets \(C \subseteq \mathbb{R}\)
3. \(f^{-1}(K)\) is compact for all compact sets \(K \subseteq \mathbb{R}\)
4. \(f^{-1}(G)\) is connected for all connected sets \(G \subseteq \mathbb{R}\).

73. Let \(A\) be an \(n \times n\) matrix, \(n \geq 2\), with characteristic polynomial \(x^{n-2}(x^2 - 1)\). Then

1. \(A^n = A^{n-2}\)
2. Rank of \(A\) is 2
3. Rank of \(A\) is at least 2
4. There exist nonzero vectors \(x\) and \(y\) such that \(A(x + y) = x - y\).

74. Let \(A, B\) and \(C\) be real \(n \times n\) matrices such that \(AB + B^2 = C\). Suppose \(C\) is nonsingular. Which of the following is always true?

1. \(A\) is nonsingular
2. \(B\) is nonsingular
3. \(A\) and \(B\) are both nonsingular
4. \(A + B\) is nonsingular.
75. Let $V$ be a real vector space and let \{ $x_1$, $x_2$, $x_3$ \} be a basis for $V$. Then

1. \{ $x_1 + x_2$, $x_2$, $x_3$ \} is a basis for $V$
2. The dimension of $V$ is 3
3. $x_1$, $x_2$, $x_3$ are pairwise orthogonal
4. \{ $x_1 - x_2$, $x_2 - x_3$, $x_1 - x_3$ \} is a basis for $V$.

76. Consider the system of $m$ linear equations in $n$ unknowns given by $Ax = b$, where $A = (a_{ij})$ is a real $m \times n$ matrix, $x$ and $b$ are $n \times 1$ column vectors. Then

1. There is at least one solution
2. There is at least one solution if $b$ is the zero vector
3. If $m = n$ and if the rank of $A$ is $n$, then there is a unique solution
4. If $m < n$ and if the rank of the augmented matrix $[A: b]$ equals the rank of $A$, then there are infinitely many solutions.

77. Let $V$ be the set of all real $n \times n$ matrices $A = (a_{ij})$ with the property that $a_{ij} = -a_{ji}$ for all $i, j = 1, 2, \cdots, n$. Then

1. $V$ is a vector space of dimension $n^2 - n$
2. For every $A$ in $V$, $a_{ii} = 0$ for all $i = 1, 2, \cdots, n$
3. $V$ consists of only diagonal matrices
4. $V$ is a vector space of dimension \( \frac{n^2 - n}{2} \).

78. Let $W$ be the set of all $3 \times 3$ real matrices $A = (a_{ij})$ with the property that $a_{ij} = 0$ if $i > j$ and $a_{ii} = 1$ for all $i$. Let $B = (b_{ij})$ be a $3 \times 3$ real matrix that satisfies $AB = BA$ for all $A$ in $W$. Then

1. Every $A$ in $W$ has an inverse which is in $W$.
2. $b_{12} = 0$
3. $b_{13} = 0$
4. $b_{23} = 0$.

79. Let $f(z)$ be an entire function with $\text{Re}(f(z)) \geq 0$ for all $z \in \mathbb{C}$. Then

1. $\text{Im}(f(z)) \geq 0$ for all $z \in \mathbb{C}$
2. $\text{Im}(f(z)) = \text{a constant}$
3. $f$ is a constant function
4. $\text{Re}(f(z)) = |z|$ for all $z \in \mathbb{C}$.

80. Let $f$ be an analytic function defined on $D = \{ z \mid |z| < 1 \}$ such that $|f(z)| \leq 1$ for all $z \in D$. Then

1. there exists $z_0 \in D$ such that $f(z_0) = 1$
2. the image of $f$ is an open set
3. $f(0) = 0$
4. f is necessarily a constant function.

81. Let \( f(z) = \frac{\sin z}{z^2} - \frac{\cos z}{z} \). Then

1. f has a pole of order 2 at \( z = 0 \)
2. f has a simple pole at \( z = 0 \)
3. \( \oint_{|z|=1} f(z) \, dz = 0 \), where the integral is taken anti-clockwise
4. the residue of f at \( z = 0 \) is \(-2\pi i\).

82. Let f be an analytic function defined on \( D = \{ z \in \mathbb{C} : |z| < 1 \} \). Then \( g : D \rightarrow \mathbb{C} \) is analytic if

1. \( g(z) = f(z) \) for all \( z \in D \)
2. \( g(z) = \overline{f(z)} \) for all \( z \in D \)
3. \( g(z) = f(\overline{z}) \) for all \( z \in D \)
4. \( g(z) = \overline{f(z)} \) for all \( z \in D \).

83. Which of the following statements involving Euler's function \( \phi \) is/are true?

1. \( \phi(n) \) is even as many times as it is odd
2. \( \phi(n) \) is odd for only two values of \( n \)
3. \( \phi(n) \) is even when \( n > 2 \)
4. \( \phi(n) \) is odd when \( n = 2 \) or \( n \) is odd.

84. Let \( p \) be a prime number and \( d \mid (p - 1) \). Then which of the following statements about the congruence \( x^d \equiv 1 \pmod{p} \) is/are true?

1. It does not have any solution
2. It has at most \( d \) incongruent solutions
3. It has exactly \( d \) incongruent solutions
4. It has at least \( d \) incongruent solutions.

85. Let \( K \) be a field, \( L \) a finite extension of \( K \) and \( M \) a finite extension of \( L \).

Then

1. \([M:K] = [M:L] + [L:K]\)
2. \([M:K] = [M:L] \cdot [L:K]\)
3. \([M:L]\) divides \([M:K]\)
4. \([L:K]\) divides \([M:K]\).

86. Let \( R \) be a commutative ring and \( R[x] \) be the polynomial ring in one variable over \( R \).

1. If \( R \) is a U.F.D., then \( R[x] \) is a U.F.D.
2. If \( R \) is a P.I.D., then \( R[x] \) is a P.I.D.
3. If \( R \) is an Euclidean domain, then \( R[x] \) is an Euclidean domain
4. If \( R \) is a field, the \( R[x] \) is an Euclidean domain.
87. Let G be a group of order 56. Then

1. All 7-Sylow subgroups of G are normal
2. All 2-Sylow Subgroups of G are normal
3. Either a 7-Sylow subgroup or a 2-Sylow subgroup of G is normal
4. There is a proper normal subgroup of G.

88. Which of the following statements is/are true?

1. 50! ends with an even number of zeros
2. 50! ends with a prime number of zeros
3. 50! ends with 10 zeros
4. 50! ends with 12 zeros.

89. Let \( X = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 = 1\} \)
\( Y = \{(x, y) \in \mathbb{R}^2 \mid |x| + |y| = 1\}, \) and
\( Z = \{(x, y) \in \mathbb{R}^2 \mid x^2 - y^2 = 1\}. \)
Then
1. \( X \) is not homeomorphic to \( Y \)
2. \( Y \) is not homeomorphic to \( Z \)
3. \( X \) is not homeomorphic to \( Z \)
4. No two of \( X, Y \) or \( Z \) are homeomorphic.

90. Let \( \tau_1, \tau_2 \) and \( \tau_3 \) be topologies on a set \( X \) such that \( \tau_1 \subseteq \tau_2 \subseteq \tau_3 \) and \( (X, \tau_2) \) is a compact Hausdorff space. Then

1. \( \tau_1 = \tau_2 \) if \( (X, \tau_1) \) is a Hausdorff space
2. \( \tau_1 = \tau_2 \) if \( (X, \tau_1) \) is a compact space
3. \( \tau_2 = \tau_3 \) if \( (X, \tau_3) \) is a Hausdorff space
4. \( \tau_2 = \tau_3 \) if \( (X, \tau_3) \) is a compact space.

91. The initial value problem \( \dot{x}(t) = 3x^{2/3}, \ x(0) = 0 \); in an interval around \( t = 0 \), has

1. no solution
2. a unique solution
3. finitely many linearly independent solutions
4. infinitely many linearly independent solutions.

92. For the system of ordinary differential equations:
\[
\frac{d}{dt} \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix},
\]

1. every solution is bounded
2. every solution is periodic
3. there exists a bounded solution
4. there exists a non periodic solution.

93. The kernel \( p(x, y) = \frac{y}{y^2 + x^2} \) is a solution of

1. the heat equation
2. the wave equation
3. the Laplace equation
4. the Lagrange equation.

94. The solution of the Laplace equation on the upper half plane, which takes the value \( \varphi(x) = e^x \) on the real line is

1. the real part of an analytic function
2. the imaginary part of an analytic function
3. the absolute value of an analytic function
4. an infinitely differentiable function.

95. Which of the following polynomials interpolate the data

\[
\begin{array}{ccc}
x & 1 & 1/2 & 3 \\
y & 3 & -10 & 2 \\
\end{array}
\]

1. \( 3 + 26(x - 1) - \frac{53}{5} (x - 1)(x - 1) \)
2. \( 3(x - 1) (x - \frac{1}{2}) -10 (x - \frac{1}{2}) (x - 3) + 10 (x - 3) (x - 1) \)
3. \( 3(x - \frac{1}{2}) (x - 3) -8 (x - 1) (x - 3) + \frac{2}{5} (x - 1)(x - \frac{1}{2}) \)
4. \( (x - 3) (x + 10) + \frac{1}{2} (x + 10) (x - 2) + 3 (x - 2) (x - 3) \).
96. The evaluation of the quantity $\sqrt{x+1} - 1$ near $x = 0$ is achieved with minimum loss of significant digits if we use the expression

1. $\sqrt{x+1} - 1$

2. $\frac{x}{\sqrt{x+1} + 1}$

3. $\left(1 - \frac{1}{\sqrt{x+1}}\right) \sqrt{x+1}$

4. $\frac{x + 2\sqrt{x+1}}{\sqrt{x+1} - 1}$

97. If $x(t)$ is an extremal of the functional $\int_a^b \left[ \frac{1}{2} m(x)^2 - cx^2 \right] dt$, where $a, b, c$ are arbitrary constants and $x = \frac{dx}{dt}$, then the function $x(t)$ satisfies

1. $m\ddot{x} + 2cx = 0$

2. $m\ddot{x} - 2cx = 0$

3. $m\dot{x}^2 + 2cx^2 = k_1$ with $k_1$ as an arbitrary constant

4. $x(t) = k_1 \sin\left(\sqrt{\frac{2c}{m}} t + k_2\right)$ with $k_1$ and $k_2$ as arbitrary constants.

98. If $u(x)$ and $v(x)$ satisfying $u(0) = 1$, $v(0) = -1$, $u(\pi/2) = 0$ and $v(\pi/2) = 0$ are the extremals of the functional $\int_0^{\pi/2} \left\{ \left(\frac{du}{dx}\right)^2 + \left(\frac{dv}{dx}\right)^2 + 2uv \right\} dx$, then

1. $u(\pi/4) + v(\pi/4) = 0$

2. $u(\pi/3) - v(\pi/3) = 0$

3. $u(\pi/4) - v(\pi/4) = 1$

4. $u(\pi/2) + v(\pi/2) = 0$.

99. Consider the integral equation $y(x) = x^2 + \lambda \int_0^1 xty(t)dt$,
where $\lambda$ is a real parameter. Then the Neumann series for the integral equation converges for all values of $\lambda$

1. except for $\lambda = 3$
2. lying in the interval $-3 < \lambda < 0$
3. lying in the interval $-3 < \lambda < 3$
4. lying in the interval $0 < \lambda < 3$.

100. The solution of the integral equation $\phi(x) = \frac{5x}{6} + \frac{1}{2} \int_0^1 xt\phi(t)dt$ satisfies

1. $\phi(0) + \phi(1) = 1$
2. $\phi\left(\frac{1}{2}\right) + \phi\left(\frac{1}{3}\right) = 1$
3. $\phi\left(\frac{1}{4}\right) + \phi\left(\frac{1}{2}\right) = 1$
4. $\phi\left(\frac{3}{4}\right) + \phi\left(\frac{1}{4}\right) = 1$.

101. A particle of unit mass is constrained to move on the plane curve $xy=1$ under gravity $g$. Then

1. the kinetic energy of the system is $\frac{1}{2}(x^2 + y^2)$
2. the potential energy of the system is $\frac{g}{x}$
3. the Lagrangian of the particle is $\frac{1}{2} x^2 \left(1 + x^{-4}\right) - \left(\frac{g}{x}\right)$
4. the Lagrangian of the particle is $\frac{1}{2} x^2 \left(1 + x^{-4}\right) + \left(\frac{g}{x}\right)$.

102. Suppose a mechanical system has the single coordinate $q$ and Lagrangian $L = \frac{1}{4} q^2 - \frac{q^2}{9}$. Then

1. the Hamiltonian is $p^2 + \left(\frac{q^2}{9}\right)$
2. Hamilton’s equations are \( \dot{q} = 2p, \ \dot{p} = -(2/9)q \)

3. \( q \) satisfies \( \ddot{q} + (4/9)q = 0 \)

4. the path in the Hamiltonian phase-space, i.e. \( q - p \) plane is an ellipse.

103. Let \( X_1, \ldots, X_n \) be i.i.d. observations from a distribution with variance \( \sigma^2 (< \infty) \). Which of the following is/are unbiased estimator(s) of \( \sigma^2 \)?

1. \( \frac{1}{n-1} \sum_{i=1}^{n} (X_i - \bar{X})^2 \)

2. \( \frac{1}{n} \sum_{i=1}^{n} (X_i - \bar{X})^2 \)

3. \( \left( \frac{n}{2} \right)^{-1} \sum_{i=1}^{n} \sum_{j<i} (X_i - X_j)^2 \)

4. \( \frac{1}{n} \sum_{i=1}^{n} X_i^2 - n\bar{X}^2 \).

104. Let \( X_1, X_2, \ldots \) be i.i.d. \( N(0,1) \) and let \( S_n = \sum_{i=1}^{n} X_i \) be the partial sums.

Which of the following is/are true?

1. \( \frac{S_n}{n} \to 0 \) almost surely

2. \( E\left( \frac{S_n}{n} \right) \to 0 \)

3. \( Var\left( \frac{S_n}{n} \right) \to 0 \)

4. \( Var\left( \frac{S_n^2}{n^2} \right) \to 0 \)

105. Let \( (X, Y) \) be a pair of independent random variables with \( X \) having exponential distribution with mean 1 and \( Y \) having uniform distribution on \( \{1, 2, \ldots, m\} \). Define \( Z = X + Y \). Then

1. \( E(Z|X) = X + \frac{m+1}{2} \)
2. \( E(Z|Y) = 1 + \frac{m+1}{2} \)

3. \( \text{Var}(Z|X) = \frac{m^2 - 1}{2} \)

4. \( \text{Var}(Z|Y) = 2. \)

106. A simple symmetric random walk on the integer line is a Markov chain which is

1. recurrent
2. null recurrent
3. irreducible
4. positive recurrent.

107. Suppose \( X \) and \( Y \) are random variables with \( E(X) = E(Y) = 0 \), \( V(X) = V(Y) = 1 \) and \( \text{Cov}(X,Y) = 0.25 \). Then which of the following is/are always true?

1. \( P\{ |X+2Y| \geq 4 \} \leq \frac{4}{16} \)
2. \( P\{ |X+2Y| \geq 4 \} \leq \frac{5}{16} \)
3. \( P\{ |X+2Y| \geq 4 \} \leq \frac{6}{16} \)
4. \( P\{ |X+2Y| \geq 4 \} \leq \frac{7}{16} \).

108. Let \( X_1, \ldots, X_n \) be a random sample from uniform \((\theta, \theta+1)\) distribution. Which of the following is/are maximum likelihood estimator(s) of \( \theta \)?

1. \( X_{(1)} \)
2. \( X_{(n)} \)
3. \( X_{(n)} - 1 \)
4. \( \frac{X_{(n)} + X_{(1)}}{2} - 0.5 \).

109. Let \( \overline{X} = (X_1, \ldots, X_n) \) be a random sample from uniform
(0. \( \theta \)). Which of the following is/are uniformly most powerful size
\( \alpha \left( 0 < \alpha < \frac{1}{2} \right) \) test(s) for testing \( H_0: \theta = \theta_0 \) against \( H_1: \theta > \theta_0 \)?

1. \( \phi_1(X) = 1, \) if \( X_{(n)} > \theta_0 \) or \( X_{(n)} < \theta_0, \alpha \frac{1}{n} = 0, \) otherwise

2. \( \phi_2(X) = 1, \) if \( X_{(n)} > \theta_0 \)

3. \( \phi_3(X) = 1, \) if \( X_{(n)} > \theta_0 \alpha \frac{1}{n} \)

4. \( \phi_4(X) = 1, \) if \( X_{(n)} < \theta_0 \frac{(\alpha / 2)}{1/n} \) or \( X_{(n)} > \theta_0 (1 - \alpha / 2)^{1/n} \)

= 0, otherwise

110. Suppose \( X_{p \times 1} \) has a \( N_p(Q, I_p) \) distribution. The distribution of \( X^T A X \) is chi-square with \( r \) degrees of freedom only if

1. A is idempotent with rank \( r \)

2. Trace (A) = Rank (A) = \( r \)

3. A is positive definite

4. A is non-negative definite with rank \( r \).

111. Let \( X_1, X_2, \ldots, X_m \) be iid random variables with common continuous cdf \( F(x) \). Also let \( Y_1, Y_2, \ldots, Y_n \) be iid random variables with common continuous cdf \( G(x) \) and \( X \)'s & \( Y \)'s are independently distributed. For testing \( H_0: F(x) = G(x) \) for all \( x \) against \( H_1: F(x) \neq G(x) \) for at least one \( x \), which of the following test is/are used?

1. Wilcoxon signed rank test

2. Kolmogorov-Smirnov test

3. Wald-Wolfowitz run test

4. Sign test.

112. Random variables \( X \) and \( Y \) are such that \( E(X) = E(Y) = 0, V(X) = V(Y) = 1, \) correlation \( (X, Y) = 0.5 \). Then the

1. conditional distribution \( Y \) given \( X = x \) is normal with mean \( 0.5x \) and variance \( 0.75 \)
2. least-squares linear regression of $Y$ on $X$ is $y=0.5x$ and of $X$ on $Y$ is $x=2y$

3. least-squares linear regression of $X$ on $Y$ is $x = 0.5y$ and of $Y$ on $X$ is $y = 2x$.

4. least-squares linear regression of $Y$ on $X$ is $y=0.5x$ and of $X$ on $Y$ is $x = 0.5y$.

113. $X$ has a binomial $(5, p)$ distribution on which an observation $x=4$ has been made. In a Bayesian approach to the estimation of $p$, a beta $(2,3)$ prior distribution (with density proportional to $p(1-p)^2$) has been formulated. Then the posterior

1. distribution of $p$ is uniform on $(0,1)$

2. mean of $p$ is $\frac{6}{10}$

3. distribution of $p$ is beta $(6,4)$

4. distribution of $p$ is binomial $(10,0.5)$.

114. In a study of voter preferences in an election, the following data were obtained

<table>
<thead>
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<th>Gender</th>
<th>Party voting for</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>C</td>
<td>B</td>
</tr>
<tr>
<td>Male</td>
<td>250</td>
<td>250</td>
</tr>
<tr>
<td>Female</td>
<td>250</td>
<td>250</td>
</tr>
<tr>
<td>Total</td>
<td>500</td>
<td>500</td>
</tr>
</tbody>
</table>

Then the

1. chi-square statistic for testing no association between party and gender is 0.

2. expected frequency under the hypothesis of no association is 250 in each cell.

3. log-linear model for cell frequency $m_{ij}$, $\log(m_{ij}) = \text{constant, i,j}=1,2$, fits perfectly to the data.

4. chi-square test of no gender-party association with 1 degree of freedom has a $p$-value of 1.
115. Let $X, Y$ and $N$ be independent random variables with $P(X=0) = \frac{1}{2} = 1 - P(X=1)$ and $Y$ following Poisson with parameter $\lambda > 0$ and $N$ following normal with mean $0$ and variance $1$. Define

$$Z = \begin{cases} Y & \text{if } X = 0 \\ N & \text{if } X = 1 \end{cases}$$

Then, the characteristic function of $Z$ is given by

1. $\left(\frac{1}{2} + \frac{1}{2}e^{it}\right)e^{-\frac{\lambda(1-e^{it})}{2} - t^2/2}$
2. $e^{-\frac{\lambda(1-e^{it})}{2} - t^2/2}$
3. $\frac{e^{-\frac{\lambda(1-e^{it})}{2} + e^{-t^2/2}}}{2}$
4. $\left(\frac{1}{2} + \frac{1}{2}e^{it}\right)\left(\frac{e^{-\frac{\lambda(1-e^{it})}{2} + e^{-t^2/2}}}{2}\right)$

116. A simple random sample of size $n$ is drawn from a finite population of $N$ units, with replacement. The probability that the $i^{th}$ ($1 \leq i \leq N$) unit is included in the sample is

1. $\frac{n}{N}$
2. $1 - \left(1 - \frac{1}{N}\right)^n$
3. $\left(\frac{N-1}{N}\right)^n$
4. $\frac{n(n-1)}{N(N-1)}$

117. Under a balanced incomplete block design with usual parameters $v, b, r, k, \lambda$, which of the following is/are true?

1. All treatment contrasts are estimable if $k \geq 2$
2. The variance of the best linear unbiased estimator of any normalized treatment contrast is a constant depending only on the design parameters and the per observation variance
3. The covariance between the best linear unbiased estimators of two mutually orthogonal treatment contrasts is strictly positive
4. The variance of the best linear unbiased estimator of an elementary treatment contrast is strictly smaller than that under a randomized block design with replication $r$. 
118. Consider a randomized (complete) block design with \( v > 2 \) treatments and \( r \geq 2 \) replicates. Which of the following statements is/are true?

1. The design is connected
2. The variance of the best linear unbiased estimator (BLUE) of every normalized treatment contrast is the same
3. The BLUE of any treatment contrast is uncorrelated with the BLUE of any contrast among replicate effects
4. The variance of the BLUE of any elementary treatment contrast is \( 2\sigma^2/r \), where \( \sigma^2 \) is the variance of an observation.

119. The starting and optimal tableaus of a minimization problem are given below. The variables are \( x_1, x_2 \) and \( x_3 \). The slack variables are \( S_1 \) and \( S_2 \).

**Starting Tableau**

<table>
<thead>
<tr>
<th></th>
<th>( Z )</th>
<th>( x_1 )</th>
<th>( x_2 )</th>
<th>( x_3 )</th>
<th>( S_1 )</th>
<th>( S_2 )</th>
<th>RHS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td>x_1</td>
<td>x_2</td>
<td>x_3</td>
<td>S_1</td>
<td>S_2</td>
<td>RHS</td>
</tr>
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<td>2</td>
<td>1</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
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<td>2</td>
<td>-1</td>
<td>0</td>
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</table>

**Optimal Tableau**

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<th>( Z )</th>
<th>( x_1 )</th>
<th>( x_2 )</th>
<th>( x_3 )</th>
<th>( S_1 )</th>
<th>( S_2 )</th>
<th>RHS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Z</td>
<td>x_1</td>
<td>x_2</td>
<td>x_3</td>
<td>S_1</td>
<td>S_2</td>
<td>RHS</td>
</tr>
<tr>
<td>x_1</td>
<td>0</td>
<td>c</td>
<td>2/3</td>
<td>2/3</td>
<td>1/3</td>
<td>0</td>
<td>e</td>
</tr>
<tr>
<td>S_2</td>
<td>0</td>
<td>d</td>
<td>8/3</td>
<td>-1/3</td>
<td>1/3</td>
<td>1</td>
<td>3</td>
</tr>
</tbody>
</table>

Which of the following are the correct values of the unknowns \( a, b, c, d \) and \( e \)

1. \( a = 2, b = 3, c = 1, d = 0, e = 2 \)
2. \( a = 2, b = -3, c = 1, d = 0, e = -2 \)
3. \( a = -2, b = 3, c = 1, d = 0, e = 2 \)
4. \( a = -2, b = 3, c = -1, d = 0, e = 2 \)

120. Consider the following linear programming problem.

Minimize \( Z = x_1 + x_2 \)

subject to \( s x_1 + t x_2 \geq 1 \)

\( x_1 \geq 0 \)

\( x_2 \) unrestricted.

The necessary and sufficient condition to make the LP

1. feasible is \( s \leq 0 \) and \( t = 0 \)
2. unbounded is \( s > t \) or \( t < 0 \)
3. have a unique solution is \( s = t \) and \( t > 0 \)
4. have a finite optimal solution is \( x_2 \geq 0 \).
अनुदेश

1. आपने हिंदी को सामान्य ज्ञान है। इस परिचय पुस्तिका में एक लोट की विभिन्न (20 भाग 'A' में + 40 भाग 'B' + 60 भाग 'C' में ) विभिन्न विकल्प प्रश्न (MCQ) दिए गए हैं। आपका भाग 'A' में से 15 विकल्प चुनें तथा भाग 'B' में से 25 प्रश्नों के उत्तर दें। यदि निर्धारित से अधिक प्रश्नों के उत्तर दिए गए तब कौनसा प्रश्न पहले भाग 'A' से 15, भाग 'B' से 25 तथा भाग 'C' से 20 उत्तरों की जानकारी जारी की जाएगी।

2. उत्तर पत्र अनुसार दिए गए हैं। अपना रोल नंबर और कृत्य का नाम लिखने के साथ पहले यह जानकारी लीजिए कि पुस्तिका में पूरा पूरा और सही है तथा कहीं के कर्ट-फट नहीं है। यदि ऐसा नहीं है तो आप इनपुट डायरेक्टर से पुस्तिका के चेतावनी के निर्देश का समर्पण करें। इसी तरह से उत्तर पत्र को भी जारी ले। इस पुस्तिका में उस काम करने के लिए अधिक प्रश्न प्राप्त करना है।

3. उत्तर पत्र के पृष्ठ 1 में दिए गए तथ्य पर अपना रोल नंबर, नाम, अपना पता तथा इस पुस्तिका पुस्तिका का क्रमांक लिखें। आपकी हस्ताक्षर भी जारी कीजिए।

4. आप अपनी आई.एम.आई. उत्तर पुस्तिका में रोल नंबर, विषय कोड, पुस्तिका कोड और कृत्य कोड से संबंधित समुचित सूचना का आवश्यक करें। जब यह एक ज्ञान परीक्षाएं की जिम्मेदारी है कि यह उत्तर पुस्तिका में दिए गए निर्देशों का पूरी साक्ष्यता से पालन करें, ऐसा न करने पर कंप्यूटर विवरणों के रखने की अनुमति नहीं कर दी जाएगी, जिसमें संपर्क आपको हानि, जिससे आपकी उत्तर पुस्तिका की अवस्थाओं की शारीरिक हो सकती है।

5. भाग 'A' में प्रतिवेद भाव 2 अंक, भाग 'B' में प्रतिवेद भाव के 3 अंक तथा भाग 'C' में प्रतिवेद भाव 4.75 अंक व भाग 'B' में प्रतिवेद भाव का ज्ञान परीक्षा भाग 'A' में @ 0.5 अंक तथा भाग 'B' में @ 0.75 अंक से जिम्मेदार है। भाग 'C' के उत्तरों के लिए ज्ञान परीक्षा भाग 'A' नहीं है।

6. भाग 'A' से भाग 'B' तक प्रतिवेद भाव के बीच अच्छी विवरण दिए गए हैं। इसमें से केवल एक विवरण ही 'सही' अथवा 'सही है' है। अपने प्रतिवेद भाव का अधिक आवश्यक विवरण दिया है। भाग 'C' में प्रतिवेद भाव का "एक" बार "एक से अधिक" विवरण नहीं हो सकते हैं। भाग 'C' में प्रतिवेद भाव के सही विवरणों का सही प्रयोग करने पर ही क्रेडिट प्राप्त होगा। तथा सही विवरणों का प्रयोग नहीं करने पर कोई अस्वीकार कोट प्रदान नहीं होगा।

7. जरूरी करने हुए या अनुसरित तरीकों का प्रयोग करने हुए पत्र आपने तथा अन्य विद्यार्थियों के लिए आवश्यक उत्तरदायी अनुसूची वह संबंधी है।

8. विद्यार्थियों को उत्तर देने के लिए सभी तरीकों से अनुसरित करने हेतु अन्य साधन जरूरी है।

9. पत्रसंदेशों के लिए परीक्षा पुस्तिका और उत्तर पत्र को इनपुट डायरेक्टर को अनुपम सीमा दीजिए।

10. कंप्यूटर का उपयोग करने की अनुमति नहीं है।

11. किसी प्रश्न में विवरण की पहलेसे अंग्रेजी संस्करण प्रबन्ध होगा।
PART A

1. The area of the shaded region in cm² is

2. The angles of a right-angled triangle shaped garden are in arithmetic progression and the smallest side is 10.00 m. The total length of the fencing of the garden in m is

3. AB is the diameter of the semicircle as shown in the diagram. If \( AQ = 2AP \) then which of the following is correct?

4. The rabbit population in community A increases at 25% per year while that in B increases at 50% per year. If the present populations of A and B are equal, the ratio of the number of the rabbits in B to that in A after 2 years will be
5. Two moles each of $O_2$ and $H_2$ are in two separate containers, each of volume $V_0$ at and 150 °C and 1 atmosphere. The two are made to react in a third container to form water vapour until $H_2$ is exhausted. When the temperature of the mixture in the third container was restored to 150 °C, its pressure became 1 atmosphere. The volume of the third container must be

1. $V_0$
2. $5V_0/4$
3. $3V_0/2$
4. $2V_0$

6. Helium and argon gases in two separate containers are at the same temperature and so have different root-mean-square (r.m.s.) velocities. The two are mixed in a third container keeping the same temperature. The r.m.s. velocity of the helium atoms in the mixture is

1. more than what it was before mixing.
2. less than what it was before mixing.
3. equal to what it was before mixing.
4. equal to that of argon atoms in the mixture.

7. The mineral talc is used in the manufacture of soap because it

(a) gives bulk to the product
(b) kills bacteria
(c) gives fragrance
(d) is soft and does not scratch the skin

Which of the above statements is/are correct?

1. (d)
2. (a) and (c)
3. (a) and (b)
4. (a) and (d)

8. 100 g of an inorganic compound $X\cdot 5H_2O$ containing a volatile impurity was kept in an oven at 150 °C for 60 minutes. The weight of the residue after heating is 8 g. The percentage of impurity in $X$ was

1. 10
2. 8
3. 20
4. 80

9. On a certain night the moon in its waning phase was a half-moon. At midnight the moon will be

1. on the eastern horizon.
2. at 45° angular height above the eastern horizon.
3. at the zenith.
4. on the western horizon.
10. A gemstone is irradiated in a nuclear reactor for 5 days. Ten days after irradiation, the activity of the chromium radioisotope in the gemstone is 600 disintegrations per hour. What is the activity of chromium radioisotope 5 days after irradiation if its half-life is 5 days?

1. 300  
2. 150  
3. 2400  
4. 1200

11. Displacement versus time curve for a body is shown in the figure. Select the graph that correctly shows the variation of the velocity with time.

12. (A) Spring balance  
    (B) Beaker with water  
    (C) Weighing machine

12. (A) Iron block  
    (B) Weighing machine  
    (C) Beaker with water
The spring balance in Fig. A reads 0.5 kg and the pan balance in Fig. B reads 3.0 kg. The iron block suspended from the spring balance is partially immersed in the water in the beaker (Fig. C). The spring balance now reads 0.4 kg. The reading on the pan balance in Fig. C is

1. 3.0 kg
2. 2.9 kg
3. 3.1 kg
4. 3.5 kg

13. The ends of a rope are fixed to two pegs, such that the rope remains slack. A pencil is placed against the rope and moved, such that the rope always remains taut. The shape of the curve traced by the pencil would be a part of

1. a circle
2. an ellipse
3. a square
4. a triangle

14. During ice skating, the blades of the ice skater's shoes exert pressure on the ice. Ice skater can efficiently skate because

1. ice gets converted to water as the pressure exerted on it increases.
2. ice gets converted to water as the pressure exerted on it decreases.
3. the density of ice in contact with the blades decreases.
4. blades do not penetrate into ice.

15. Four sedimentary rocks A, B, C and D are intruded by an igneous rock R as shown in the cross-section diagram. Which of the following is correct about their ages?

1. A is the youngest followed by B, C, D and R.
2. R is the youngest followed by A, B, C and D.
3. D is the youngest followed by C, B, A and R.
4. A is the youngest followed by R, B, C and D.
16. The strain in a solid subjected to continuous stress is plotted.

Which of the following statements is true?

1. The solid deforms elastically till the point of failure.
2. The solid deforms plastically till the point of failure.
3. The solid comes back to original shape and size on failure.
4. The solid is permanently deformed on failure.

17. Growth of an organism was monitored at regular intervals of time, and is shown in the graph below. Around which time is the rate of growth zero?

1. Close to day 10
2. On day 20
3. Between days 20 and 30
4. Between days 30 and 40

18. A Tall plant with Red seeds (both dominant traits) was crossed with a dwarf plant with white seeds. If the segregating progeny produced equal number of tall red and dwarf white plants, what would be the genotype of the parents?

1. TtRr × TtRR
2. TtRr × tttr
3. TTRR × tttr
4. TtRR × TtRr
19. Three sunflower plants were placed in conditions as indicated below.

- Plant A: still air
- Plant B: moderately turbulent air
- Plant C: still air in the dark

Which of the following statements is correct?

1. Transpiration rate of plant B > that of plant A.
2. Transpiration rate of plant A > that of plant B.
3. Transpiration rate of plant C = that of plant A.
4. Transpiration rate of plant C > that of plant A > that of plant B.

20. Which of the following is indicated by the accompanying diagram?

1. \( a + ab + ab^2 + \ldots = a/(1-b) \) for \( |b|<1 \)
2. \( a > b \) implies \( a^3 > b^3 \)
3. \( (a+b)^2 = a^2 + 2ab + b^2 \)
4. \( a > b \) implies \( -a < -b \)
21. और 'MATHEMATICS' के अक्षरों के क्रमबद्ध से
किरदार बनाये जा सकते हैं?
1. 5040 2. 4989600
3. 111 4. 8!

22. 50,000 के धारावाहिक भाजक कितने हैं?
1. 20 2. 30
3. 40 4. 50

23. यदि A और B दो nxn यालायिक आकृत हैं। तभी क्या कहा कि?
1. ज्ञाति (A+B) = ज्ञाति (A) + ज्ञाति (B).
2. ज्ञाति (A+B) ≤ ज्ञाति (A) + ज्ञाति (B).
3. ज्ञाति (A+B) = सूत्रतम {ज्ञाति (A), ज्ञाति (B)}.
4. ज्ञाति (A+B) = महत्तम {ज्ञाति (A), ज्ञाति (B)}.

24. यदि कि
\[ f_n(x) = \begin{cases} 1 - nx, & \text{for } x \in [0, \frac{1}{n}] \\ 0, & \text{for } x \in \left[\frac{1}{n}, 1\right] \end{cases} \]

1. \( \lim_{n \to \infty} f_n(x), [0, 1] \) पर सांतत्य फलन की परिभाषा
करता है।
2. \( f_n \), [0, 1] पर एककाल काल अभिग्रहित होता है।
3. सभी \( x \in [0, 1] \) के लिए \( \lim_{n \to \infty} f_n(x) = 0 \) है।
4. सभी \( x \in [0, 1] \) के लिए \( \lim_{n \to \infty} f_n(x) \) का अवरोध
है।

25. संख्या \( \sqrt{2e^{\pi}} \) एक
1. परिमेय संख्या है।
2. अभिग्रहित है।
3. अपरिमेय संख्या है।
4. अवरोधक संख्या है।

21. The number of words that can be formed by
permuting the letters of 'MATHEMATICS' is
1. 5040 2. 4989600
3. 111 4. 8!

22. The number of positive divisors of 50,000 is
1. 20 2. 30
3. 40 4. 50

23. Let A, B be nxn real matrices. Which of the
following statements is correct?
1. rank (A+B) = rank (A) + rank (B).
2. rank (A+B) ≤ rank (A) + rank (B).
3. rank (A+B) = min {rank (A), rank (B)}.
4. rank (A+B) = max {rank (A), rank (B)}.

24. Let \( f_n(x) = \begin{cases} 1 - nx, & \text{for } x \in [0, \frac{1}{n}] \\ 0, & \text{for } x \in \left[\frac{1}{n}, 1\right] \end{cases} \)

Then
1. \( \lim_{n \to \infty} f_n(x) \) defines a continuous function on
   [0, 1].
2. \( \{f_n\} \) converges uniformly on [0, 1].
3. \( \lim_{n \to \infty} f_n(x) = 0 \) for all \( x \in [0, 1] \).
4. \( \lim_{n \to \infty} f_n(x) \) exists for all \( x \in [0, 1] \).

25. The number \( \sqrt{2e^{\pi}} \) is
1. a rational number.
2. a transcendental number.
3. an irrational number.
4. an imaginary number.
26. Let $\zeta$ be a primitive cube root of unity. Define

$$A = \begin{pmatrix} \zeta^{-1} & 0 \\ 0 & \zeta \end{pmatrix}.$$ 

For a vector $v = (v_1, v_2, v_3) \in \mathbb{R}^3$ define

$$|v|_A = \sqrt{v^t A v}$$

where $v^t$ is transpose of $v$. If $w = (1,1,1)$ then $|w|_A$ equals

1. 0
2. 1
3. $-1$
4. 2

27. Let $M = \{(a_1, a_2, a_3) : a_3 \in \{1, 2, 3, 4\}, a_1 + a_2 + a_3 = 6\}$. The number of elements in $M$ is

1. 8
2. 9
3. 10
4. 12

28. The last digit of $38^{3001}$ is

1. 6
2. 2
3. 4
4. 8

29. The dimension of the vector space of all symmetric matrices $A = (a_{ij})$ of order $n \times n$ ($n \geq 2$) with real entries, $a_{11} = 0$ and trace zero is

1. $(n^2 - n - 4)/2$
2. $(n^2 + n - 4)/2$
3. $(n^2 - n - 3)/2$
4. $(n^2 + n - 3)/2$

30. Let $I = [0,1] \subset \mathbb{R}$. For $x \in \mathbb{R}$, let $\phi(x) = \text{dist} (x, I) = \inf \{|x-y| : y \in I\}$. Then

1. $\phi(x)$ is discontinuous somewhere on $\mathbb{R}$.
2. $\phi(x)$ is continuous on $\mathbb{R}$ but not continuously differentiable exactly at $x = 0$.
3. $\phi(x)$ is continuous on $\mathbb{R}$ but not continuously differentiable exactly at $x = 0$ and at $x = 1$.
4. $\phi(x)$ is differentiable on $\mathbb{R}$. 

26. मानने कि $\zeta$ एक का आदिन चर समूह है / $A$ की परिभाषा है : 

$$A = \begin{pmatrix} \zeta^{-1} & 0 \\ 0 & \zeta \end{pmatrix}.$$ 

सत्स $v = (v_1, v_2, v_3) \in \mathbb{R}^3$ के लिए $|v|_A$ की परिभाषा है $\sqrt{v^t A v}$ जहाँ $v^t$, $v$ का परिवर्तन है / अगर $w = (1,1,1)$ तो $|w|_A$

1. 0 के समान है /
2. 1 के समान है /
3. $-1$ के समान है /
4. 2 के समान है /

27. मानने कि $M = \{(a_1, a_2, a_3) : a_3 \in \{1, 2, 3, 4\}, a_1 + a_2 + a_3 = 6\}$. $M$ के अंकों की संख्या है

1. 8
2. 9
3. 10
4. 12

28. $(38)^{3001}$ का अंतिम अंक है / 

1. 6
2. 2
3. 4
4. 8

29. सांख्यिक प्रविष्टि या शृंखला अनुसार वाले $n \times n$ ($n \geq 2$) के रूपों संगमण आकर्षित $A = (a_{ij})$, $a_{11} = 0$ के लिए $A$ की अवधि है :

1. $(n^2 - n - 4)/2$
2. $(n^2 + n - 4)/2$
3. $(n^2 - n - 3)/2$
4. $(n^2 + n - 3)/2$

30. मानने कि $I = [0,1] \subset \mathbb{R}$. $x \in \mathbb{R}$ के साथ मानने कि 

$\phi(x) = \text{dist} (x, I) = \inf \{|x-y| : y \in I\}$ / तो

1. $\mathbb{R}$ पर कसी $\phi(x)$ अस्तित्व है /
2. $\mathbb{R}$ पर $\phi(x)$ संतत है परन्तु यथार्थता $x = 0$ पर 
3. $\mathbb{R}$ पर $\phi(x)$ संतत है परन्तु यथार्थता $x = 0$ पर $x = 1$ पर संतत असंतत नहीं है /
4. $\mathbb{R}$ पर $\phi(x)$ असंतत है /
31. Let \( a_n = \sin \pi/n \). For the sequence \( a_1, a_2, \ldots \) the supremum is

1. 0 is and it is attained.
2. 0 and it is not attained.
3. 1 and it is attained.
4. 1 and it is not attained.

32. Let \( \sum_{n=1}^{\infty} \frac{1}{n^2} = \pi^2 / 6 \) be a known result. Let \( \sum_{n=1}^{\infty} \frac{1}{(2n+1)^2} \) be another result.

\[
\begin{align*}
1. & \quad \frac{\pi^2}{12} \\
2. & \quad \frac{\pi^2}{12} - 1 \\
3. & \quad \frac{\pi^2}{8} \\
4. & \quad \frac{\pi^2}{8} - 1
\end{align*}
\]

33. Let \( f(x, y) = u(x, y) + i v(x, y) \) be a complex-valued function of the form \( f(z) = u(x, y) + i v(x, y) \).

Suppose that \( u(x, y) = 3x^2y \).

Then

1. \( f \) cannot be holomorphic on \( \mathbb{C} \) for any choice of \( v \).
2. \( f \) is holomorphic on \( \mathbb{C} \) for a suitable choice of \( v \).
3. \( f \) is holomorphic on \( \mathbb{C} \) for all choices of \( v \).
4. \( u \) is not differentiable.

34. Let \( f: \mathbb{R}^2 \times \mathbb{R}^2 \rightarrow \mathbb{R} \) be a bilinear map, i.e., linear in each variable separately. Then for \( (V, W) \in \mathbb{R}^2 \times \mathbb{R}^2 \), the derivative \( Df(V, W) \) evaluated at \( (H, K) \in \mathbb{R}^2 \times \mathbb{R}^2 \) is given by

\[
\begin{align*}
1. & \quad f(V, K) + f(H, W) \\
2. & \quad f(H, K) \\
3. & \quad f(V, H) + f(W, K) \\
4. & \quad f(H, V) + f(W, K)
\end{align*}
\]

35. Let \( N \) be the vector space of all real polynomials of degree at most 3. Define

\[
S: N \rightarrow N \text{ by } (Sp)(x) = p(x + 1), \quad p \in N.
\]

Then the matrix of \( S \) in the basis \( \{1, x, x^2, x^3\} \), considered as column vectors, is given by:
36. Let $F$ be a field of 8 elements and $A = \{ x \in F \mid x^2 = 1 \text{ and } x^k = 1 \text{ for all natural numbers } k < 7 \}$. Then the number of elements in $A$ is

1. 1
2. 2
3. 3
4. 6

37. The power series $\sum_{n=0}^{\infty} (z-1)^n$ converges if

1. $|z| \leq 3$
2. $|z| < \sqrt{3}$
3. $|z-1| < \sqrt{3}$
4. $|z-1| \leq \sqrt{3}$

38. Consider the group $G = \mathbb{Q}/\mathbb{Z}$ where $\mathbb{Q}$ and $\mathbb{Z}$ are the groups of rational numbers and integers respectively. Let $n$ be a positive integer. Then is there a cyclic subgroup of order $n$?

1. not necessarily.
2. yes, a unique one.
3. yes, but not necessarily a unique one.
4. never

39. Let $f(x) = x^3 + 2x^2 + 1$ and $g(x) = 2x^2 + x + 2$ over $\mathbb{Z}_5$.

1. $f(x)$ and $g(x)$ are irreducible
2. $f(x)$ is irreducible, but $g(x)$ is not.
3. $g(x)$ is irreducible, but $f(x)$ is not.
4. neither $f(x)$ nor $g(x)$ is irreducible.
40. The number of non-trivial ring homomorphisms from $\mathbb{Z}_{12}$ to $\mathbb{Z}_{28}$ is

1. 1
2. 3
3. 4
4. 7

41. Consider the initial value problem

$$y'(t) = f(t) y(t), \quad y(0) = 1$$

where $f: \mathbb{R} \to \mathbb{R}$ is continuous. Then this initial value problem has

1. infinitely many solutions for some $f$.
2. a unique solution in $\mathbb{R}$.
3. no solution in $\mathbb{R}$ for some $f$.
4. a solution in an interval containing 0, but not on $\mathbb{R}$ for some $f$.

42. Let $V$ be the set of all bounded solutions of the ODE

$$u''(t) - 4u'(t) + 3u(t) = 0, \quad t \in \mathbb{R}$$

Then $V$

1. is a real vector space of dimension 2.
2. is a real vector space of dimension 1.
3. contains only the trivial function $u \equiv 0$.
4. contains exactly two functions.

43. The function

$$u(x,t) = \begin{cases} \frac{1}{\sqrt{t}} e^{-\frac{x^2}{4t}} & , \quad t > 0, \ x \in \mathbb{R} \\ 0 & , \quad t \leq 0, \ x \in \mathbb{R} \end{cases}$$

is a solution of the heat equation in

1. $\{x, t\} : x \in \mathbb{R}, \ t \in \mathbb{R}$.
2. $\{x, t\} : x \in \mathbb{R}, \ t > 0$ but not in the set $\{x, t\} : x \in \mathbb{R}, \ t < 0$.
3. $\{x, t\} : x \in \mathbb{R}, \ t \in \mathbb{R} \setminus \{(0,0)\}$.
4. $\{x, t\} : x \in \mathbb{R}, \ t > -1$. 
44. The second order PDE
\[ u_{yy} - yu_{xx} + x^2 u = 0 \]
is
1. elliptic for all \( x \in \mathbb{R}, y \in \mathbb{R} \).
2. parabolic for all \( x \in \mathbb{R}, y \in \mathbb{R} \).
3. elliptic for all \( x \in \mathbb{R}, y < 0 \).
4. hyperbolic for all \( x \in \mathbb{R}, y < 0 \).

45. Consider a second order ordinary differential Equation (ODE) and its finite difference representation. Identify which of the following statements is correct.

1. The finite difference representation is unique.
2. The finite difference representation is unique for some ODE.
3. There is no unique finite difference scheme for the ODE.
4. The uniqueness of a finite difference scheme can not be determined.

46. The variational problem of extremizing the functional
\[ I(y(x)) = \int_{a}^{b} y^2(x) dx \quad y'(b) = 1 \]
has
1. a unique solution.
2. exactly two solutions.
3. an infinite number of solutions.
4. no solution.

47. For the linear integral equation
\[ \phi(x) = x + \int_{0}^{1} \phi(\xi) d\xi, \]
the resolvent kernel \( R(x, \xi; 1) \) is
1. \( 1/2 \)
2. 2
3. 3/2
4. 4

48. If the Hamiltonian of a dynamical system is given by \( H = p^2 - q^2 \), then as \( t \to \infty \)
1. \( q \to \infty, p \to \infty \)
2. \( q \to 0, p \to 0 \)
3. \( q \to \infty, p \to 0 \)
4. \( q \to 0, p \to \infty \)
49. संख्यात्मक बंटन प्रभाव F₁(t) एवं F₂(t) एवं भविष्यवाणी प्रभाव f₁(t) एवं f₂(t) के दो जोड़के तरीके द्वारा T₁ एवं T₂ की जोड़के भविष्यवाणी ग्रंथित है: h₁(t) = 3t² एवं h₂(t) = 4t³, t > 0 है। तो

1. सभी t > 0 के लिए F₁(t) ≥ F₂(t).
2. सभी t > 1 के लिए F₁(t) < F₂(t).
3. E(T₁) < E(T₂).
4. सभी t > 0 के लिए f₁(t) < f₂(t).

50. माना कि \(X₁, X₂, \ldots \sim N(1,1)\) के अनुसार सर्वाधिक प्रभाव स्वतंत्र लूप के बाद यादृच्छिक बाल हैं। माना कि \(n \geq 1\) के लिए \(S_n = X₁² + X₂² + \cdots + X_n²\) है। तो प्रभाव \(S_n\) का स्वरूप है:

\[
\lim_{n \to \infty} \frac{S_n}{n} = \sigma^2
\]

1. 4
2. 6
3. 1
4. 0

51. माना कि \(\{X_n : n \geq 0\}\) एक परिमित अवरोध समावेश S पर स्वतंत्र स्वरूप प्रभावित आवृत्त बाल एक मार्कोव अनुसंधान है। माना कि सर्वाधिक अल्पवक्तावासी नहीं है। तद्वारा मार्कोव अनुसंधान

1. के अन्तर्गत विश्वसनीय स्वतंत्र बाल है।
2. का एक ही अन्तर्गत स्वतंत्र बाल है।
3. का कोई भी स्वतंत्र बाल नहीं है।
4. के लीक-लीक दो स्वतंत्र बाल नहीं है।

52. माना कि \(X\) एवं \(Y\) दो स्वतंत्र यादृच्छिक बाल हैं, जहाँ 0 के \(X\) के \(Y\) के \(X = Y\) समान है। माना कि \(U = X + Y\) एवं \(V = X - Y\) हो।

1. \(U \oplus V\) हमेशा स्वतंत्र है।
2. \(U \oplus V\) दोनों का बंटन समान है।
3. \(U\) हमेशा \(0\) के \(Y\) के \(X = Y\) समान है।
4. \(V\) हमेशा \(0\) के \(Y\) के \(X = Y\) समान है।

53. एक नीचे दो सामान्यिक पत्तों के प्रति मतदाताओं की पसंद की आंकानीय सूचना \(2 \times 2\) तालिका में लिखनुसार संगठित है। सभी कथन को पहलाने जा सकते हैं:

<table>
<thead>
<tr>
<th>सूचना</th>
<th>पत्ता A</th>
<th>पत्ता B</th>
<th>पत्ता C</th>
<th>कुल</th>
</tr>
</thead>
<tbody>
<tr>
<td>पत्ता A</td>
<td>200</td>
<td>400</td>
<td>600</td>
<td>1200</td>
</tr>
<tr>
<td>पत्ता B</td>
<td>100</td>
<td>300</td>
<td>400</td>
<td>800</td>
</tr>
<tr>
<td>पत्ता C</td>
<td>300</td>
<td>700</td>
<td>1000</td>
<td>2000</td>
</tr>
</tbody>
</table>

61. दो जोड़के तरीके द्वारा \(T₁\) एवं \(T₂\) के दो जोड़के तरीके द्वारा T₁ एवं T₂ की जोड़के भविष्यवाणी ग्रंथित है: h₁(t) = 3t² एवं h₂(t) = 4t³, t > 0 है। तो

1. \(F₁(t) \geq F₂(t)\) के लिए \(t > 0\) है।
2. \(F₁(t) < F₂(t)\) के लिए \(t > 1\) है।
3. \(E(T₁) < E(T₂)\) है।
4. \(f₁(t) < f₂(t)\) के लिए \(t > 0\) है।

50. Let \(X₁, X₂, \ldots \sim i.i.d. N(1,1)\) random variables. Let \(S_n = X₁² + X₂² + \cdots + X_n²\) for \(n \geq 1\). Then \(\lim_{n \to \infty} \frac{\text{Var}(S_n)}{n} = \sigma^2\)

1. 4
2. 6
3. 1
4. 0

51. Let \(\{X_n : n \geq 0\}\) be a Markov chain on a finite state space \(S\) with stationary transition probability matrix. Suppose that the chain is not irreducible. Then the Markov chain

1. admits infinitely many stationary distributions.
2. admits a unique stationary distribution.
3. may not admit any stationary distribution.
4. cannot admit exactly two stationary distributions.

52. Suppose \(X\) and \(Y\) are independent random variables where \(Y\) is symmetric about \(0\). Let \(U = X + Y\) and \(V = X - Y\). Then

1. \(U\) and \(V\) are always independent.
2. \(U\) and \(V\) have the same distribution.
3. \(U\) is always symmetric about \(0\).
4. \(V\) is always symmetric about \(0\).

53. Consider the following \(2 \times 2\) table of frequencies of voter preferences to two parties classified by gender, in an election. Identify the correct statement:

<table>
<thead>
<tr>
<th>Gender</th>
<th>Party A</th>
<th>Party B</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>200</td>
<td>400</td>
<td>600</td>
</tr>
<tr>
<td>Female</td>
<td>100</td>
<td>300</td>
<td>400</td>
</tr>
<tr>
<td>Total</td>
<td>300</td>
<td>700</td>
<td>1000</td>
</tr>
</tbody>
</table>
1. If there is no association between party and gender, the expected frequencies are

<table>
<thead>
<tr>
<th>Party</th>
<th>Male</th>
<th>Female</th>
</tr>
</thead>
<tbody>
<tr>
<td>180</td>
<td>420</td>
<td>120</td>
</tr>
<tr>
<td>280</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. The chi-square statistic for testing no association is 0.
3. Gender and party are not associated.
4. Both males and females equally prefer party C.

54. Let $X_1, X_2, \ldots, X_n$ be $n \geq 2$ i.i.d. observations from $N(\mu, \sigma^2)$ distribution, where $-\infty < \mu < \infty$ and $0 < \sigma^2 < \infty$ are unknown parameters. Let $\hat{\sigma}^2_{MLE}$ and $\hat{\sigma}^2_{UMVUE}$ denote the maximum likelihood and uniformly minimum variance unbiased estimates of $\sigma^2$ respectively. Identify the correct statement:

1. $\hat{\sigma}^2_{MLE}$ has the same variance as that of $\hat{\sigma}^2_{UMVUE}$.
2. $\hat{\sigma}^2_{MLE}$ has larger variance than that of $\hat{\sigma}^2_{UMVUE}$.
3. $\hat{\sigma}^2_{MLE}$ has smaller mean squared error than that of $\hat{\sigma}^2_{UMVUE}$.
4. $\hat{\sigma}^2_{MLE}$ and $\hat{\sigma}^2_{UMVUE}$ have the same mean squared error.

55. Suppose that we have i.i.d. observations $X_1, X_2, \ldots, X_n$ with a normal distribution. Suppose further that we have an independent set of observations $Y_1, Y_2, \ldots, Y_n$ which are also i.i.d. with the same normal distribution. Let $R_X = \text{the sum of the ranks of the } X \text{s when they are ranked in the combined set of } X \text{ and } Y \text{ values}$, and $R_Y = \text{the sum of the ranks of the } Y \text{s in the combined set. Then}$

1. $P(R_X - R_Y > 0) > \frac{1}{2}$.
2. $P(R_Y - R_X > 0) > \frac{1}{2}$.
3. $E(R_X) = E(R_Y)$.
4. $P(R_Y = R_X) = 1$.

56. Consider a simple linear regression model

$$Y_i = \beta X_i + \epsilon$$

Let $\hat{Y}_{i0}$ be the least squares predictor of $Y$ at $X = x_0$ based on $n$ observations $(Y_i, X_i), i = 1, \ldots, n$ and $\bar{X} = \frac{1}{n} \sum_{i=1}^{n} X_i$.

Then the standard error of the predictor $\hat{Y}_{i0}$
1. decreases as \( x_0 \) moves away from \( \bar{X} \).
2. increases as \( x_0 \) moves away from \( \bar{X} \).
3. increases as \( x_0 \) moves closer to 0.
4. decreases as \( x_0 \) moves closer to 0.

57. A box contains \( N \) tickets which are numbered 1, 2, \( \cdots \), \( N \). The value of \( N \) is however, unknown. A simple random sample of \( n \) tickets is drawn without replacement from the box. Let \( X_1, X_2, \ldots, X_n \) be numbers on the tickets obtained in the \( 1^{\text{st}} \), \( 2^{\text{nd}} \), \( \ldots \), \( n^{\text{th}} \) draws respectively. Which of the following is an unbiased estimator of \( N \)?

1. \( 2\bar{X} - 1 \) where \( \bar{X} = \frac{1}{N} (X_1 + \ldots + X_n) \)
2. \( 2\bar{X} + 1 \)
3. \( 2\bar{X} + \frac{1}{2} \)
4. \( 2\bar{X} - \frac{1}{2} \)

58. In a clinical trial \( n \) randomly chosen persons were enrolled to examine whether two different skin creams, A and B, have different effects on the human body. Cream A was applied to one of the randomly chosen arms of each person, cream B to the other arm. Which statistical test is to be used to examine the difference? Assume that the response measured is a continuous variable.

1. Two-sample t-test if normality can be assumed.
2. Paired t-test if normality can be assumed.
3. Two-sample Kolmogorov-Smirnov test.
4. Test for randomness.

59. Suppose that the variables \( x_1 \geq 0 \) and \( x_2 \geq 0 \) satisfy the constraints \( x_1 + x_2 \geq 3 \) and \( x_1 + 2x_2 \geq 4 \). Which of the following is true?

1. The maximum value of \( 5x_1 + 7x_2 \) is 21 and it does not have any finite minimum.
2. The minimum value of \( 5x_1 + 7x_2 \) is 17 and it does not have any finite maximum.
3. The maximum value of \( 5x_1 + 7x_2 \) is 21 and its minimum value is 17.
4. \( 5x_1 + 7x_2 \) neither has a finite maximum nor a finite minimum.
60. Let $X(t)$ be the number of customers in an M/M/1 queuing system with arrival rate $\lambda > 0$ and service rate $\mu > 0$. The process $X(t)$ is a

1. Poisson process with rate $\lambda - \mu$.
2. pure birth process with birth rate $\lambda - \mu$.
3. birth and death process with birth rate $\lambda$ and death rate $\mu$.
4. birth and death process with birth rate $\frac{1}{\lambda}$ and death rate $\frac{1}{\mu}$.

60. अवश्य मान्यता $\lambda > 0$ तथा अवश्य मान्यता $\mu > 0$ के एक M/M/1 कार्यालय प्रणाली में मानने कि $X(t)$ प्रारूपों की संख्या है। प्रक्रिया $X(t)$ एक

1. मान्यता $\lambda - \mu$ का प्रारूप प्रक्रिया है।
2. जनन मान्यता $\lambda - \mu$ की शुद्ध जनन प्रक्रिया है।
3. जनन मान्यता $\lambda$ तथा मृत्यु मान्यता $\mu$ की जनन-मृत्यु प्रक्रिया है।
4. जनन मान्यता $\frac{1}{\lambda}$ तथा मृत्यु मान्यता $\frac{1}{\mu}$ की जनन-मृत्यु प्रक्रिया है।
61. Consider the function

\[ f(x) = \cos(|x - 5|) + \sin(|x - 3|) + |x + 10|^3 - (|x| + 4)^2. \]

At which of the following points is \( f \) not differentiable?

1. \( x = 5 \)  
2. \( x = 3 \)  
3. \( x = -10 \)  
4. \( x = 0 \)

62. \( \mathbb{R}^2 \) के निम्न उपसमुच्चायों ने कौन से समुच्चय हैं?

1. \( \{(x, y) : |x| \leq 1, |y| \geq 2\} \)
2. \( \{(x, y) : |x| \leq 1, |y|^2 \leq 2\} \)
3. \( \{(x, y) : x^2 + 3y^2 \leq 5\} \)
4. \( \{(x, y) : x^2 \leq y^2 + 5\} \)

63. \( f \) और \( g \) ने जो \( C = \{f : [0, 1] \to \mathbb{R} \} \) के एक संख्यात्मक फलन हैं, \( d(f, g) \) है?

1. \( d(f, g) = \sup \{|f(x) - g(x)| : x \in [0,1]\} \).
2. \( d(f, g) = \inf \{|f(x) - g(x)| : x \in [0,1]\} \).
3. \( d(f, g) = \int_0^1 |f(x) - g(x)|\,dx \).
4. \( d(f, g) = \sup \{|f(x) \cdot g(x)| : x \in [0,1]\} + \int_0^1 |f(x) - g(x)|\,dx \).
63. Which of the following are metrics on \( C = \{ f: [0, 1] \rightarrow \mathbb{R} \text{ is a continuous function} \} \):

1. \( d(f, g) = \sup \{| f(x) - g(x) | : x \in [0, 1] \} \).
2. \( d(f, g) = \inf \{| f(x) - g(x) | : x \in [0, 1] \} \).
3. \( d(f, g) = \int_0^1 | f(x) - g(x) | \, dx \).
4. \( d(f, g) = \sup \{| f(x) - g(x) | : x \in [0, 1] \} + \int_0^1 | f(x) - g(x) | \, dx \).

64. For each \( j = 1, 2, 3, \ldots \), let \( A_j \) be a finite set containing at least two distinct elements. Then

1. \( \bigcup_{j=1}^{\infty} A_j \) is a countable set.
2. \( \bigcup_{n=1}^{\infty} \bigcap_{j=1}^{n} A_j \) is uncountable.
3. \( \bigcap_{j=1}^{\infty} A_j \) is uncountable.
4. \( \bigcup_{n=1}^{\infty} \bigcup_{j=1}^{n} A_j \) is uncountable.

65. Which of the following is/are correct?

1. \( \left(1 + \frac{1}{n} \right)^{n+1} \rightarrow e \text{ as } n \rightarrow \infty \).
2. \( \left(1 + \frac{1}{n+1} \right)^n \rightarrow e \text{ as } n \rightarrow \infty \).
3. \( \left(1 + \frac{1}{n} \right)^{n^2} \rightarrow e \text{ as } n \rightarrow \infty \).
4. \( \left(1 + \frac{1}{n^2} \right)^n \rightarrow e \text{ as } n \rightarrow \infty \).
66. Which of the following is/are true?

1. \( \log \frac{x+y}{2} \leq \log x + \log y \) for all \( x, y > 0 \).
2. \( e^{\frac{x+y}{2}} \leq e^x + e^y \) for all \( x, y > 0 \).
3. \( \sin \frac{x+y}{2} \leq \frac{\sin x + \sin y}{2} \) for all \( x, y > 0 \).
4. \( \frac{(x+y)^k}{2^k} \leq \max \{ x^k, y^k \} \) for all \( x, y > 0 \) and all \( k \geq 1 \).

67. Let \( f: [a, b] \to \mathbb{R} \) be a measurable function. Then

1. If \( \int_a^d f(x)dx = 0 \) for all \( a \leq c < d \leq b \) then \( f = 0 \) a.e.
2. If \( \int_c^e f(x)dx = 0 \) for all \( a \leq c \leq b \), then \( f = 0 \) a.e.
3. If \( \int_{a}^{d} f(x)dx = 0 \) for all \( a \leq c < d \leq b \), does not necessarily imply that \( f = 0 \) a.e.

4. If \( \int_{a}^{d} f(x)dx = 0 \) for all \( a \leq c \leq b \) does not necessarily imply that \( f = 0 \) a.e.

68. \( \mathbb{R}^n \) \( \iff x = (x_1, x_2, \ldots, x_n) \) \( \iff y = (y_1, y_2, \ldots, y_n) \). 1 \( \leq p < \infty \) के लिए माने कि

\[
d_p(x, y) = \left( \sum_{i=1}^{n} |x_i - y_i|^p \right)^{1/p}
\]

\( d_p(x, y) = \text{उत्तरम} \{ |x_j - y_j| : j = 1, 2, \ldots, n \} \) है। मानें कि \( B_p = \{ x \in \mathbb{R}^n : d_p(x, 0) < 1 \} \), 1 \( \leq p \leq \infty \). निम्न में से कौन सा / से कौन सा है / है?

1. \( d_\infty \)-मौलिक में \( B_1 \) विभाजित है।
2. \( d_\infty \)-मौलिक में \( B_2 \) विभाजित है।
3. \( d_2 \)-मौलिक में \( B_1 \) विभाजित नहीं है।
4. \( d_2 \)-मौलिक में \( B_2 \) विभाजित नहीं है।

68. For \( x = (x_1, x_2, \ldots, x_n) \) and \( y = (y_1, y_2, \ldots, y_n) \) in \( \mathbb{R}^n \) let \( d_p(x, y) = \left( \sum_{i=1}^{n} |x_j - y_j|^p \right)^{1/p} \) for \( 1 \leq p < \infty \), and \( d_p(x, y) = \max \{ |x_j - y_j| : j = 1, 2, \ldots, n \} \).

Let \( B_p = \{ x \in \mathbb{R}^n : d_p(x, 0) < 1 \} \), 1 \( \leq p \leq \infty \).

Which of the following are correct?

1. \( B_1 \) is open in the \( d_\infty \)-metric.
2. \( B_1 \) is open in the \( d_\infty \)-metric.
3. \( B_1 \) is not open in the \( d_2 \)-metric.
4. \( B_1 \) is not open in the \( d_2 \)-metric.

69. \( f(x, y) = (7x + x^4, 3x + 4y + y^4) \) द्वारा परिभाषित फलन \( f : \mathbb{R}^2 \rightarrow \mathbb{R}^2 \) पर विचारें।

1. \((0, 0)\) पर \( f \) असंतत है।
2. \((0, 0)\) पर \( f \) संतत है बि \((0, 0)\) पर सभी डिरेक्टर-अक्षा के अनुप्रवाह है।
3. \((0, 0)\) पर \( f \) अवकलनीय है बि \( अवकलन \( Df(0,0) \) अवकलकलीय नहीं है।
4. \((0, 0)\) पर \( f \) अवकलनीय है बि \( अवकलन \( Df(0,0) \) भी अवकलकलीय नहीं है।

69. Consider the map \( f : \mathbb{R}^2 \rightarrow \mathbb{R}^2 \) defined by

\( f(x, y) = (7x + x^4, 3x + 4y + y^4). \)

Then

1. \( f \) is discontinuous at \((0,0)\).
2. \( f \) is continuous at \((0,0)\) and all directional derivatives exist at \((0,0)\).
3. \( f \) is differentiable at \((0,0)\) but the derivative \( Df(0,0) \) is not invertible.
4. \( f \) is differentiable at \((0,0)\) and the derivative \( Df(0,0) \) is invertible.
70. \[ C[0, 1] \] is a complete metric space with respect to the norm:

1. \[ \|f\|_\infty := \sup \{|f(x)| : x \in [0, 1]\}. \]
2. \[ \|f\|_1 := \int_0^1 |f(x)| \, dx. \]
3. \[ \|f\|_{0,1} := \|f\|_\infty + |f(1)| + |f(0)|. \]
4. \[ \|f\|_2 = \sqrt{\int_0^1 |f(x)|^2 \, dx}. \]

71. The space \( C[0, 1] \) of continuous functions on \([0, 1]\) is complete with respect to the norm.

1. \[ \|f\|_\infty := \sup \{|f(x)| : x \in [0, 1]\}. \]
2. \[ \|f\|_1 := \int_0^1 |f(x)| \, dx. \]
3. \[ \|f\|_{0,1} := \|f\|_\infty + |f(1)| + |f(0)|. \]
4. \[ \|f\|_2 = \sqrt{\int_0^1 |f(x)|^2 \, dx}. \]

71. \( D_{(a,b)}(r) = \{(x, y) : (x-a)^2 + (y-b)^2 < r\} \) is a complete metric space. Which of the following subsets of \( \mathbb{R} \) are connected?

1. \( D_{(0,0)}(1) \cup \{(1, 0)\} \cup D_{(2,0)}(1) \)
2. \( D_{(0,0)}(1) \cup D_{(2,0)}(1) \)
3. \( D_{(0,0)}(1) \cup \{(1, 0)\} \cup D_{(0,2)}(1) \)
4. \( D_{(0,0)}(1) \cup D_{(0,2)}(1) \)

72. \( X = \{x \in [0,1] : x = 1/n, n \in \mathbb{N}\} \) is a complete metric space. Which of the following subsets of \( \mathbb{R} \) are connected?

1. \( X \) is connected when it is connected (1)
2. \( X \) is not connected when it is not connected (2)
3. \( X \) is connected when it is connected (3)
4. \( X \) is not connected when it is not connected (4)
72. Let \( X = \{ x \in [0,1] : x \times 1/n, n \in \mathbb{N} \} \) be given the subspace topology. Then

1. \( X \) is connected but not compact.
2. \( X \) is neither compact nor connected.
3. \( X \) is compact and connected.
4. \( X \) is compact but not connected.

73. रिम आयामों में से कौन से धारात्मक-निर्विरोध हैं?

1. \[
\begin{bmatrix}
2 & 1 \\
1 & 2
\end{bmatrix}
\]
2. \[
\begin{bmatrix}
1 & 2 \\
2 & 1
\end{bmatrix}
\]
3. \[
\begin{bmatrix}
4 & -1 \\
-1 & 4
\end{bmatrix}
\]
4. \[
\begin{bmatrix}
0 & 4 \\
4 & 0
\end{bmatrix}
\]

73. Which of the following matrices are positive definite?

1. \[
\begin{bmatrix}
2 & 1 \\
1 & 2
\end{bmatrix}
\]
2. \[
\begin{bmatrix}
1 & 2 \\
2 & 1
\end{bmatrix}
\]
3. \[
\begin{bmatrix}
4 & -1 \\
-1 & 4
\end{bmatrix}
\]
4. \[
\begin{bmatrix}
0 & 4 \\
4 & 0
\end{bmatrix}
\]

74. नामें कि \( n \)-विंग्रीय कार्यात्मक सार्वजनिक समाधेत \( V \) एक बुल्कीर अभिन्नता समाधेत है। नामें कि उपसमाधेत \( V_0 \subset V, A \) को अन्तर्गत \( V \) का प्रमाणित है। नामें कि \( k = \text{विभ} \ (V_0) < n \) एवं नामें कि कुछ \( \lambda \in \mathbb{R} \) के लिए \( A^2 = \lambda A \) हो

1. \( \lambda = 1 \).
2. \( \text{सार्वलक} \ A = |\lambda| \).
3. \( \lambda, A \) का एक मात्र अभिन्नता समाधेत है।
4. \( \text{एक अनुच्छेद उपसमाधेत } V_1 \subset V \) है ताकि लगभग \( x \in V_1 \) के लिए \( Ax = 0 \)

74. Let \( A \) be a non-zero linear transformation on a real vector space \( V \) of dimension \( n \). Let the subspace \( V_0 \subset V \) be the image of \( V \) under \( A \). Let \( k = \text{dim } V_0 < n \) and suppose that for some \( \lambda \in \mathbb{R}, A^2 = \lambda A \). Then

1. \( \lambda = 1 \).
2. \( \det A = |\lambda| \).
3. \( \lambda \) is the only eigenvalue of \( A \).
4. There is a nontrivial subspace \( V_1 \subset V \) such that \( Ax = 0 \) for all \( x \in V_1 \).
75. Let $C$ be a $n \times n$ real matrix. Let $W$ be the vector space spanned by $\{I, C, C^2, \ldots, C^{2n}\}$. The dimension of the vector space $W$ is

1. $2n$
2. at most $n$
3. $n^2$
4. at most $2n$

75. Let $V_1$ and $V_2$ be subspaces of a vector space $V$. Which of the following is necessarily a subspace of $V$?

1. $V_1 \cap V_2$
2. $V_1 \cup V_2$
3. $V_1 + V_2 = \{x + y : x \in V_1, y \in V_2\}$
4. $V_1 \setminus V_2 = \{x \in V_1$ and $y \in V_2\}$

76. Let $N$ be a nonzero $3 \times 3$ matrix with the property $N^2 = 0$. Which of the following is/are true?

1. $N$ is not similar to a diagonal matrix.
2. $N$ is similar to a diagonal matrix.
3. $N$ has one non-zero eigenvector.
4. $N$ has three linearly independent eigenvectors.
Let \( x, y \in \mathbb{C}^n \). Consider \( f(x, y) = \sup_{\theta, \varphi} \left\| e^{i\theta} x + e^{i\varphi} y \right\| : \theta, \varphi \in \mathbb{R} \). Which of the following is/are correct?

1. \( f(x, y) \leq \|x\|^2 + \|y\|^2 + 2\|x, y\| \)
2. \( f(x, y) = \|x\|^2 + \|y\|^2 + 2 \Re\{x, y\} \)
3. \( f(x, y) = \|x\|^2 + \|y\|^2 + 2\|x, y\| \)
4. \( f(x, y) > \|x\|^2 + \|y\|^2 + 2\|x, y\| \)

"""Eकक II/Unit II"

79. निम्न में से कौन-से समूह \( C[0, 1] \) में दर्शन है? (उस्का-उसका समत्वता के विषय में \( [0, 1] \) पर वास्तविक मूल्यों वाले संतत फलनों की समता)

1. \( \{f \in C[0, 1] : f \text{ एक बहुपद है} \} \)
2. \( \{f \in C[0, 1] : f(0) = 0\} \)
3. \( \{f \in C[0, 1] : f(0) \neq 0\} \)
4. \( \{f \in C[0, 1] : \int_0^1 f(x) \, dx = 5\} \)

79. Which of the following sets are dense in \( C[0, 1] \) (the space of real valued continuous functions on \( [0, 1] \) with respect to sup-norm topology)?

1. \( \{f \in C[0, 1] : f \text{ is a polynomial}\} \)
2. \( \{f \in C[0, 1] : f(0) = 0\} \)
3. \( \{f \in C[0, 1] : f(0) \neq 0\} \)
4. \( \{f \in C[0, 1] : \int_0^1 f(x) \, dx = 5\} \)

80. माना कि \( f : \mathbb{C} \to \mathbb{C}, n \geq 1 \) के लिए \( f \left( \frac{1}{n} \right) = \frac{n}{2n+1} \) का लगातार करता हुआ एक नेरोमफिक कन्पख है जो \( 0 \) पर विलोमक है।

1. \( f(0) = 1/2 \)
2. \( z = -2 \) पर \( f \) का एक संकरण अन्तिक है।
3. \( f(2) = 1/4 \)
4. ऐसा कोई नेरोमफिक कन्पख का अस्तित्व नहीं है।

80. Let \( f : \mathbb{C} \to \mathbb{C} \) be a meromorphic function analytic at \( 0 \) satisfying \( f \left( \frac{1}{n} \right) = \frac{n}{2n+1} \) for \( n \geq 1 \).

Then

1. \( f(0) = 1/2 \)
2. \( f \) has a simple pole at \( z = -2 \)
3. \( f(2) = 1/4 \)
4. no such meromorphic function exists
81. Let $f$ be an entire function. If $\text{Im } f \geq 0$, then

1. $f$ is constant
2. $f$ is constant
3. $f = 0$
4. $f'$ is a nonzero constant

82. Let $f : \mathbb{D} \rightarrow \mathbb{D}$ be holomorphic with $f(0) = 0$ and $f'(1/2) = 0$, where $\mathbb{D} = \{z : |z| < 1\}$. Which of the following statements are correct?

1. $|f'(1/2)| \leq 4/3$
2. $|f'(0)| \leq 1$
3. $|f'(1/2)| \leq 4/3$ and $|f'(0)| \leq 1$
4. $f(z) = z, z \in \mathbb{D}$

83. $z = x + iy$ द्वारा $z \in \mathbb{C}$ के लिये अन्वेषित करें:

$H^+ = \{z \in \mathbb{C} : y > 0\}$
$H^- = \{z \in \mathbb{C} : y < 0\}$
$L^+ = \{z \in \mathbb{C} : x > 0\}$
$L^- = \{z \in \mathbb{C} : x < 0\}$

फलन $f(z) = \frac{2z + 1}{5z + 3}$

1. $H^+$ को $H^+$ के ऊपर व $H^-$ को $H^-$ के ऊपर प्रतिविद्धित करता है
2. $H^+$ को $H^+$ के ऊपर व $H^-$ को $H^-$ के ऊपर प्रतिविद्धित करता है
3. $H^+$ को $L^+$ के ऊपर व $H^-$ को $L^-$ के ऊपर प्रतिविद्धित करता है
4. $H^+$ को $L^+$ के ऊपर व $H^-$ को $L^-$ के ऊपर प्रतिविद्धित करता है
83. For \( z \in \mathbb{C} \) of the form \( z = x + iy \), define
\[
\mathbb{H}' = \{ z \in \mathbb{C} : y > 0 \},
\]
\[
\mathbb{H} = \{ z \in \mathbb{C} : y < 0 \},
\]
\[
\mathbb{L}' = \{ z \in \mathbb{C} : x > 0 \},
\]
\[
\mathbb{L} = \{ z \in \mathbb{C} : x < 0 \}.
\]

The function \( f(z) = \frac{2z + 1}{5z + 3} \)

1. maps \( \mathbb{H}' \) onto \( \mathbb{H}' \) and \( \mathbb{H} \) onto \( \mathbb{H} \).
2. maps \( \mathbb{H}' \) onto \( \mathbb{H} \) and \( \mathbb{H} \) onto \( \mathbb{H}' \).
3. maps \( \mathbb{H}' \) onto \( \mathbb{L}' \) and \( \mathbb{L} \) onto \( \mathbb{L}' \).
4. maps \( \mathbb{H} \) onto \( \mathbb{L} \) and \( \mathbb{L} \) onto \( \mathbb{H} \).

84. \( z = 0 \) पर फलन \( f(z) = \exp \left( \frac{z}{1 - \cos z} \right) \) का

1. एक अपमेय विविधता है।
2. एक सदिश है।
3. एक अविनय विविधता है।
4. \( z = 0 \) के आस पास \( f(z) \) का लॉरेंट्स विश्लेषण के अन्तर्गत नहीं है।

84. At \( z = 0 \), the function \( f(z) = \exp \left( \frac{z}{1 - \cos z} \right) \) has

1. a removable singularity.
2. a pole.
3. an essential singularity.
4. the Laurent expansion of \( f(z) \) around \( z = 0 \) has infinitely many positive and negative powers of \( z \).

85. यदि \( R = \mathbb{Q}[x]/I \) जहाँ \( I, 1 + x^2 \) द्वारा जनित एक आदर्श है। माना कि \( R \) में \( x \) का \( x \) सहायक \( y \) है। तो \( R \) पर \( x \) अवस्थापक है।

1. \( y^2 + 1 \)
2. \( y^2 + y + 1 \)
3. \( y^2 - y + 1 \)
4. \( y^3 + y^2 + y + 1 \)

85. Let \( R = \mathbb{Q}[x]/I \) where \( I \) is the ideal generated by \( 1 + x^2 \). Let \( y \) to the coset of \( x \) in \( R \). Then

1. \( y^2 + 1 \) is irreducible over \( R \).
2. \( y^2 + y + 1 \) is irreducible over \( R \).
3. \( y^2 - y + 1 \) is irreducible over \( R \).
4. \( y^3 + y^2 + y + 1 \) is irreducible over \( R \).
86. निम्न में से कोन-सा सही है?

1. \( \sin^7, \mathbb{Q} \) पर बीजीय है।
2. \( \cos \pi/17, \mathbb{Q} \) पर बीजीय है।
3. \( \sin^{-1}, \mathbb{Q} \) पर बीजीय है।
4. \( \sqrt{2 + \sqrt{\pi}}, \mathbb{Q}(\pi) \) पर बीजीय है।

86. Which of the following is true?

1. \( \sin^7 \) is algebraic over \( \mathbb{Q} \).
2. \( \cos \pi/17 \) is algebraic over \( \mathbb{Q} \).
3. \( \sin^{-1} \) is algebraic over \( \mathbb{Q} \).
4. \( \sqrt{2 + \sqrt{\pi}} \) is algebraic over \( \mathbb{Q}(\pi) \).

87. सारे यह \( f(x) = x^3 + x^2 + x + 1 \) और \( g(x) = x^3 + 1 \) \( \mathbb{Q}[x] \) में होते हैं?

1. यह \( f(x), g(x) \) का योग है \( x + 1 \).
2. यह \( f(x), g(x) \) का तपाई है \( x^3 - 1 \).
3. लघुतम सहातत्त्व \( f(x), g(x) \) का योग है \( x^2 + x^2 + x^2 + 1 \).
4. लघुतम सहातत्त्व \( f(x), g(x) \) का तपाई है \( x^3 + x^3 + x^2 + 1 \).

87. Let \( f(x) = x^3 + x^2 + x + 1 \) and \( g(x) = x^3 + 1 \). Then in \( \mathbb{Q}[x] \),

1. \( \gcd(f(x), g(x)) = x + 1 \).
2. \( \gcd(f(x), g(x)) = x^3 - 1 \).
3. \( \text{lcm}(f(x), g(x)) = x^2 + x^2 + x^2 + 1 \).
4. \( \text{lcm}(f(x), g(x)) = x^3 + x^3 + x^2 + 1 \).

88. कोटि 36 के कोई समूह \( G \) व उसके उपसमूह \( H \) जो \( G \) कोटि 4 के लिये

1. \( H \subset Z(G) \).
2. \( H = Z(G) \).
3. \( G \) हो \( H \) प्रसामान्य है।
4. \( H \) एक आबेली समूह है।

88. For any group \( G \) of order 36 and any subgroup \( H \) of \( G \) order 4,

1. \( H \subset Z(G) \).
2. \( H = Z(G) \).
3. \( H \) is normal in \( G \).
4. \( H \) is an abelian group.

89. नामने कि \( G \) समूह \( S_5 \times S_3 \) का गिरनिर्देश करता है।

1. \( G \) का 2-पैलो उपसमूह प्रसामान्य है।
2. \( G \) का 3-पैलो उपसमूह प्रसामान्य है।
3. \( G \) का एक अनुसूची प्रसामान्य उपसमूह है।
4. \( G \) का एक प्रसामान्य उपसमूह कोटि 72 का है।
89. Let $G$ denote the group $S_4 \times S_3$. Then

1. a 2-Sylow subgroup of $G$ is normal.
2. a 3-Sylow subgroup of $G$ is normal.
3. $G$ has a nontrivial normal subgroup.
4. $G$ has a normal subgroup of order 72.

90. मानने कि $X$ एक प्रस्तावना हाउस्डोफ़ अन्तरगत्ति है | मानने कि $A_1, A_2, A_3, X$ के संबंध मध्यम मूल्यहू जो पुनःस्थान
अन्तर्गत्ति है | तो $X$ पर एक सतह वातस्तिक सूचकाला फलन $f$ हमेशा रहता है ताकि $f(x) = a_i$ यदि $x \in A_i$,
$i = 1, 2, 3$.

1. हर $a_i$ या तो 0 या 1 होने पर ही है।
2. कम से कम $a_1, a_2, a_3$ में कोई भी दो संबंधी के समान होने पर ही है।
3. $a_1, a_2, a_3$ के सभी वातस्तिक सूचकाला के लिए है।
4. तभी जब सबसे न्यूनतम $A_1, A_2$ एवं $A_3$ में से एक रिखा है।

90. Let $X$ be a normal Hausdorff space. Let $A_1, A_2, A_3$ be closed subsets of $X$ which are pairwise
disjoint. Then there always exists a continuous real valued function $f$ on $X$ such that

\[ f(x) = a_i \text{ if } x \in A_i, \quad i = 1, 2, 3 \]

1. iff each $a_i$ is either 0 or 1.
2. iff at least two of the numbers $a_1, a_2, a_3$ are equal.
3. for all real values of $a_1, a_2, a_3$.
4. only if one among the sets $A_1, A_2$ and $A_3$ is empty.

एकक III/Unit III

91. समाधान अधिक समीकरण प्रश्नाली $\frac{d}{dx} Y = AY$. $Y(0) = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$. तब क्रियाएं, जहाँ $A = \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}$

\[ Y = \begin{bmatrix} y_1(x) \\ y_2(x) \end{bmatrix} \]

1. $y_1(x) \to \infty$ एवं $y_2(x) \to 0$ जब $x \to \infty$.
2. $y_1(x) \to 0$ एवं $y_2(x) \to 0$ जब $x \to \infty$.
3. $y_1(x) \to \infty$ एवं $y_2(x) \to -\infty$ जब $x \to -\infty$.
4. $y_1(x), y_2(x) \to -\infty$ जब $x \to -\infty$.

91. Consider the system of ODE

\[ \frac{d}{dx} Y = AY, \quad Y(0) = \begin{bmatrix} 2 \\ -1 \end{bmatrix} \]

where $A = \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}$ and $Y = \begin{bmatrix} y_1(x) \\ y_2(x) \end{bmatrix}$. Then

1. $y_1(x) \to \infty$ and $y_2(x) \to 0$ as $x \to \infty$.
2. $y_1(x) \to 0$ and $y_2(x) \to 0$ as $x \to \infty$. 
92. For the boundary value problem

\[ y'' + \lambda y = 0; \ y(0) = 0, \ y(1) = 0, \]

there exists an eigenvalue \( \lambda \) for which there corresponds an eigenfunction in \( (0, 1) \) that

1. does not change sign.
2. changes sign.
3. is positive.
4. is negative.

93. The solution of the boundary value problem

\[ \frac{d^2 y}{dx^2} + y = \csc x; \quad 0 < x < \frac{\pi}{2}, \ y(0) = 0, \quad y\left(\frac{\pi}{2}\right) = 0 \]

is

1. convex
2. concave
3. negative
4. positive

94. For the problem

\[ xu_x + yu_y = 0 \]

subject to

\[ u(x, y) = x, \quad x^2 + y^2 = 1 \text{ on the circle} \]

1. \( \text{for all } x, y \in \mathbb{R} \) the problem has a solution.
2. \( \{ (x, y) \in \mathbb{R}^2 : (x, y) \neq (0, 0) \} \) is an open set with a solution.
3. \( \{ (x, y) \in \mathbb{R}^2 : (x, y) \neq (0, 0) \} \) is a domain with a solution.
4. \( \{ (x, y) \in \mathbb{R}^2 : (x, y) \neq (0, 0) \} \) is not a domain with a solution, and hence the problem has no solution.
94. The Cauchy problem

\[
\begin{align*}
\frac{\partial u}{\partial t} + ju_x &= 0 \\
u(x, y) &= x, \quad \text{on } x^2 + y^2 = 1
\end{align*}
\]

has

1. a solution for all \(x \in \mathbb{R}, y \in \mathbb{R}\).
2. an unique solution in \(\{(x, y) \in \mathbb{R}^2 : (x, y) \neq (0, 0)\}\)
3. a bounded solution in \(\{(x, y) \in \mathbb{R}^2 : (x, y) \neq (0, 0)\}\)
4. an unique solution in \(\{(x, y) \in \mathbb{R}^2 : (x, y) \neq (0, 0)\}\), but the solution is unbounded.

95. \(\text{माने कि } u \text{ क्रम समीकरण}

\[
\begin{align*}
u_t - u_{xx} &= 0, \quad 0 < x < \pi \text{ and } t > 0 \\
u(0, t) &= u(\pi, t) = 0, \quad t > 0 \\
u(x, 0) &= \sin x + \sin 2x, \quad 0 \leq x \leq \pi
\end{align*}
\]

का एक हल है। तो

1. \(\text{राशि } x \in (0, \pi) \text{ के } \text{लिए } u(x, t) \to 0 \text{ जब } t \to \infty\)
2. \(\text{राशि } x \in (0, \pi) \text{ के } \text{लिए } t' u(x, t) \to 0 \text{ जब } t \to \infty\)
3. \(x \in (0, \pi), t > 0 \text{ के } \text{लिए } e' u(x, t) \text{ एक परिस्थित फलन है।}\)
4. \(\text{राशि } x \in (0, \pi) \text{ के } \text{लिए } e^{nt} u(x, t) \to 0 \text{ जब } t \to \infty\)

95. Let \(u\) be a solution of the heat equation

\[
\begin{align*}
u_t - u_{xx} &= 0, \quad 0 < x < \pi \text{ and } t > 0 \\
u(0, t) &= u(\pi, t) = 0, \quad t > 0 \\
u(x, 0) &= \sin x + \sin 2x, \quad 0 \leq x \leq \pi
\end{align*}
\]

Then

1. \(u(x, t) \to 0 \text{ as } t \to \infty \text{ for all } x \in (0, \pi).\)
2. \(t' u(x, t) \to 0 \text{ as } t \to \infty \text{ for all } x \in (0, \pi).\)
3. \(e' u(x, t) \text{ is a bounded function for } x \in (0, \pi), t > 0.\)
4. \(e^{nt} u(x, t) \to 0 \text{ as } t \to \infty \text{ for all } x \in (0, \pi).\)

96. \(\text{माने कि } u \text{ परिवर्ती गण्ड्रा समीकरण}

\[
\begin{align*}
\dot{u} + \frac{1}{t} u' &= f(t), \quad t \in (0, 1) \\
u' (0) &= a, \quad u(1) = b
\end{align*}
\]
Let \( u \) be a solution of the boundary value problem

\[
\begin{align*}
u'' + \frac{1}{t} u' &= f(t), & t \in (0,1) \\
u'(0) &= a, & u(1) = b
\end{align*}
\]

Define for \( x^2 + y^2 \leq 1 \), \( v(x, y) = u\left(\sqrt{x^2 + y^2}\right) \) and \( g(x, y) = f\left(\sqrt{x^2 + y^2}\right) \), then \( v \) is a solution of the PDE

\[
\begin{align*}
v_{xx} + v_{yy} &= g & \text{in } \{ (x, y) : x^2 + y^2 < 1 \} \\
v(x, y) &= 0 & \text{on } \{ (x, y) : x^2 + y^2 = 1 \}
\end{align*}
\]

1. \( a > 0 \) and \( b > 0 \)
2. \( a > 0 \) and \( b = 0 \)
3. \( a = 0 \) and \( b = 0 \)
4. \( a < 0 \) and \( b = 0 \)
3. UTM में भागावत की जा सकती है प्रश्नोंकर्ता या उपरि विकल्पमय प्रश्नोंकर्ता सूचना है नियम है।
4. UTM में भागावत की जा सकती है एवं UTM का हल (1) का भी हल है।

97. Given that an upper triangular matrix (UTM) is invertible if and only if all its diagonal elements are different from zero, consider the linear system

\[2x_1 + 3x_2 - x_3 = 5 \]
\[4x_1 + 4x_2 - 3x_3 = 3 \]
\[-2x_1 + 3x_2 - x_3 = 1\] (1)

Then system (1) can be transformed into an UTM but is not invertible because the diagonal entries of the UTM are not different from zero.

98. Let \( f(x) = x^2 - x - 2 = 0 \) \( \ldots \) (1) be the equation. Now, let \( x = g(x) \) where \( g(x) \) is the function of the equation (1). Then, \( g(x) \) is the solution of (1).

98. Consider the function

\[ f(x) = x^2 - x - 2 = 0 \] (1)

Let \( x = g(x) \), so that any fixed point of \( g(x) \) is a solution of (1). Then

1. \( g(x) = x - \frac{x^2 - x - 2}{m} \), \( m \in [-a, a] \) is a possible choice where \( a \) is positive constant.
2. \( g(x) = x^2 - 2 \), \( g(x) = 1 + \frac{2}{x} \) are possible choices.
3. \( g(x) = x - \frac{x^2 - x - 2}{K} \), \( K \neq 0 \), \( K \in \mathbb{R} \) is a possible choice.
4. \( g(x) = x^2 - 2 \), \( g(x) = 1 + \frac{2}{x} \) are the only possible choices.
The integral equation

\[ \phi(x) = \lambda \int_0^x K(x, \xi) \phi(\xi) d\xi \]

where \( \lambda \) is a parameter, and \( K(x, \xi) \) is the integral kernel.

leads to a boundary value problem \( \phi''(x) - f(\lambda) \phi(x) = 0, \phi(0) = 0, \phi(\pi) = 0 \), where \( f(\lambda) \) is known. Then the boundary value problem has

1. a unique solution when \( f(\lambda) = 0 \).
2. infinite number of solutions when \( f(\lambda) > 0 \).
3. no solution when \( f(\lambda) < 0 \).
4. a unique solution when \( \lambda > 1 \).

Kritisik

\[ I(z(x, y)) = \iint_D \left( \frac{\partial^2 z}{\partial x^2} \right)^2 + \left( \frac{\partial^2 z}{\partial y^2} \right)^2 - 2z \]
dxdy. जहां \( D \) एक बौद्धिक है जिसकी बिंदुओं पर \(-1 \leq x \leq 1, -1 \leq y \leq 1, \) एवं \( z = 0 \) है के विभागीकरण की समस्या का समन्वयित हल \( z = z_0(x, y) \) इस प्रकार हैं:

1. \( z_0 = \sum_{i=1}^n \alpha_i \phi_i(x, y) \), जहाँ \( \alpha_i \) विश्वास है एवं \( D \) पर फलन \( \phi_i \) एकमात्र स्वतंत्र है।
2. \( z_0 = \alpha_1 \phi_1(x, y) + \alpha_2 \phi_2(x, y) \), जहाँ \( \alpha_1 \) एवं \( \alpha_2 \) विश्वास है एवं \( \phi_1 \) एवं \( \phi_2 \) के संदर्भ आशिक अपकल होते हैं।
3. \( z_0 = \alpha_0 \phi(x, y) \) जहाँ \( \alpha_0 \) विश्वास है एवं \( D \) पर \( \phi \) संदर्भ है।
4. \( z_0 = (x^2 - 1)(y^2 - 1)/16. \)
100. An approximate solution \( z = z_0 (x, y) \) to the problem of extremizing the functional

\[
I(z(x, y)) = \iint_D \left[ \left( \frac{\partial z}{\partial x} \right)^2 + \left( \frac{\partial z}{\partial y} \right)^2 - 2z \right] dx dy,
\]

where \( D \) is the square, \(-1 \leq x \leq 1, -1 \leq y \leq 1 \), and \( z = 0 \) on the boundary of the square, is of the form

1. \( z_0 = \sum_{i=1}^{n} \alpha_i \phi_i (x, y) \), where \( \alpha_i \) are constants and functions \( \phi_i \) are linearly independent in \( D \).

2. \( z_0 = \alpha \phi (x, y) + \alpha_0 \phi_0 (x, y) \), where \( \alpha \) and \( \alpha_0 \) are constants, and \( \phi \) and \( \phi_0 \) have continuous partial derivatives.

3. \( z_0 = \alpha \phi (x, y) \) where \( \alpha \) is a constant and \( \phi \) is continuous in \( D \).

4. \( z_0 = (x^2 - 1)(y^2 - 1)/16 \).

101. निम्न ते कोणा सत्य सत्यी है/है?

1. हैमिल्टन नियम दिल्मान नियम का अनुप्रयोग है।
2. हम ताक आपकी कृतियों के आवकोश के बीच से संबंध दिया नहीं जाता व हैमिल्टन नियम लोगों के लिये लागू नहीं होता।
3. हैमिल्टन नियम लघुत्र समीकरणों का अनुप्रयोग है।
4. न्यूटन का हस्तिय नियम हैमिल्टन नियम का अनुप्रयोग है।

101. Which of the following is/are correct?

1. Hamilton’s principle follows from the D’Alembert’s principle.
2. Hamilton’s principle is not usually applicable to nonholonomic system, unless a relation connecting the differentials of generalized coordinates is given.
3. Hamilton’s principle follows from Lagrange’s equations.
4. Newton’s second law of motion follows from the Hamilton’s principle.

102. यदि \( L \) एक फलन \( f \) है तो

1. उपरीत \( L \) का फलन \( f \) के \( \partial L/\partial f \) \( \partial f/\partial L \) समीकरण है।
2. समीकरण \( L \) के \( \partial L/\partial x \) \( \partial x/\partial L \) संख्या की संख्या रूप से समान है।
3. उपरीत \( L \) के \( \partial L/\partial f \) \( \partial f/\partial L \) रूप का \( \partial L/\partial f \) संख्या है।
4. जब \( \partial L/\partial f \) में उफ्फतित है, उपरीत फलन \( L \) का \( \partial L/\partial f \) एक एक दोहरीय क्षण है।

102. Let \( L \) denote the Lagrangian of a system. Then the

1. Lagrange’s equations are second order differential equations.
2. Total number of equations is equal to the number of generalized coordinates.
3. Lagrangian \( L \) is not unique in its functional form, but the form of the Lagrange’s equation of motion can be preserved.
4. Lagrangian function is a quadratic function of generalized velocity when the potential exists.
103. Let \( F(x, y) \), \( G(x) \) and \( H(y) \) be the joint c.d.f. of \((X, Y)\), marginal c.d.f. of \(X\) and marginal c.d.f. of \(Y\) respectively. Define:

\[
U = \begin{cases} 
1 & \text{if } X \leq a \\
-1 & \text{if } X > a 
\end{cases} \quad \text{and} \quad V = \begin{cases} 
1 & \text{if } Y \leq b \\
-1 & \text{if } Y > b 
\end{cases}
\]

where \( a \) and \( b \) are fixed real numbers. Then

1. If \( \text{Cov}(U, V) = 0 \) then \( F(x, y) = G(x) H(y) \) for all \( x \) and \( y \).
2. If \( F(x, y) = G(x) H(y) \) for all \( x \) and \( y \) then \( \text{Cov}(U, V) = 0 \).
3. If \( U \) and \( V \) are independent then \( X \) and \( Y \) are independent.
4. If \( X \) and \( Y \) are independent then \( U \) and \( V \) are independent.

104. Which of the following conditions imply independence of the random variables \( X \) and \( Y \)?

1. \( p(X > a \mid Y > a) = p(X > a) \) for all \( a \in \mathbb{R} \).
2. \( p(X > a \mid Y < b) = p(X > a) \) for all \( a, b \in \mathbb{R} \).
3. X and Y are uncorrelated.
4. \( E[(X - a)(Y - b)] = E(X - a) \cdot E(Y - b) \) for all \( a, b \in \mathbb{R} \).

105. Consider a Markov chain with state space \( S = \{1,2,3,4,5\} \) and stationary transition probability matrix \( P \) given by

\[
\begin{pmatrix}
0.1 & 0 & 0.2 & 0.7 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0.7 & 0 & 0.1 & 0.2 & 0 \\
0.2 & 0 & 0.7 & 0.1 & 0 \\
0 & 0.5 & 0 & 0 & 0.5
\end{pmatrix}
\]

Let \( p_{ij}^{(n)} \) be the \((i,j)\)th element of \( P^n \)

Then

1. \( \sum_{n=1}^{\infty} p_{ij}^{(n)} = 1 \).
2. (0.25, 0.25, 0.25, 0.25, 0) is a stationary distribution for the Markov chain.
Let \( g(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \) for \( x \in \mathbb{R} \) and \( u \) be a continuous function on \( \mathbb{R} \) such that:

(i) \( u(-x) = -u(x) \), for all \( x \in \mathbb{R} \), and \( u \) non-zero

(ii) \( u(x) = 0 \) for \( x \notin (-1, 1) \),

(iii) \( |u(x)| \leq \frac{1}{2\sqrt{2\pi}} \), for all \( x \in \mathbb{R} \).

Let \( f(x) = g(x) + u(x) \), for all \( x \in \mathbb{R} \). Then

1. \( f \) can take negative values.
2. \( f(x) > 0 \) for all \( x \) and \( f \) is not integrable.
3. \( f \) is a probability density function on \( \mathbb{R} \).
4. \( f \) is an integrable function.
107. Let $X_1, X_2, \ldots$ be independent random variables with $X_n$ being uniformly distributed between $-n$ and $3n$, $n = 1, 2, \ldots$.
Let $S_N = \frac{1}{\sqrt{N}} \sum_{n=1}^{N} X_n$ for $N = 1, 2, \ldots$ and let $F_N$ be the distribution function of $S_N$. Also let $\Phi$ denote the distribution function of a standard normal random variable. Which of the following is/are true?

1. $\lim_{N \to \infty} F_N(0) \leq \Phi(0)$
2. $\lim_{N \to \infty} F_N(0) \geq \Phi(0)$
3. $\lim_{N \to \infty} F_N(1) \leq \Phi(1)$
4. $\lim_{N \to \infty} F_N(1) \geq \Phi(1)$

108. यदि $X_1$ और $X_2$ स्वतंत्र हैं एवं उनके घनाश्चम $f_1(x) = \frac{1}{\theta} e^{-\frac{x}{\theta}}, x > 0$ और $f_2(x) = \frac{2}{\theta} e^{-2x/\theta}, x > 0$ हैं।

1. $X_1 + X_2$, $\theta$ के लिए पर्याप्त है ।
2. $X_1 + 2X_2$, $\theta$ के लिए पर्याप्त है ।
3. $X_1 + 2X_2$, $\theta$ के लिए सुरक्षित है ।
4. $\frac{1}{2} (X_1 + 2X_2)$, $\theta$ के लिए अनपर्याप्त है ।

108. Suppose $X_1$ has density $f_1(x) = \frac{1}{\theta} e^{-\frac{x}{\theta}}, x > 0$ and $X_2$ has density $f_2(x) = \frac{2}{\theta} e^{-2x/\theta}, x > 0$ and $X_1, X_2$ are independent. Then

1. $X_1 + X_2$ is sufficient for $\theta$.
2. $X_1 + 2X_2$ is sufficient for $\theta$.
3. $X_1 + 2X_2$ is complete for $\theta$.
4. $\frac{1}{2} (X_1 + 2X_2)$ is unbiased for $\theta$.

109. यदि हमारे पास $n (\geq 2)$ एक समानता-समान रूप से मिलता प्रक्षेप $X_1, X_2, \ldots, X_n$ है, तो एक एक आन्तरिक निर्देश त्रिकोण $\mu, \sigma^2$ बिंदुओं के साथ, जहाँ $-\infty < \mu < \infty$ एवं $0 < \sigma^2 < \infty$ दोनों अनपर्याप्त हैं।

1. $\sigma^2$ का अधिकांक संगमिता अकलन $\sigma^2$ का अनपर्याप्त आकलन है।
2. $\sigma^2$ का प्रसरण असंगमिता अकलन की $\sigma^2$ के अधिकांक संगमिता अकलन की तुलना में एक बड़ा और आकलन है।
3. $\sigma^2$ के अधिकांक संगमिता अकलन एवं प्रसरण अकलन, दोनों उपयोगी संगमता आकलन है।
4. $\sigma^2$ के किसी भी अनपर्याप्त अकलन के लिए $\sigma^2$ का एक दृस्ता आकलन है जिसकी वर्ग माध्य मूल्य पर्याप्त करने।
109. Suppose that we have \( n \geq 2 \) i.i.d. observations \( X_1, X_2, \ldots, X_n \) each with a common \( N(\mu, \sigma^2) \) distribution, where \(-\infty < \mu < \infty\) and \( 0 < \sigma^2 < \infty \) are both unknown. Then

1. the maximum likelihood estimate of \( \sigma^2 \) is an unbiased estimate for \( \sigma^2 \).
2. the uniformly minimum variance unbiased estimate of \( \sigma^2 \) has smaller mean squared error than the maximum likelihood estimate of \( \sigma^2 \).
3. both the maximum likelihood estimate and the uniformly minimum variance estimate of \( \sigma^2 \) are asymptotically consistent estimates.
4. for any unbiased estimate of \( \sigma^2 \), there is another estimate of \( \sigma^2 \) with a smaller mean squared error.

110. यदि आपके द्वारा \( X_1, X_2, \ldots, X_{25} \) अवलोकन \( [\theta - \frac{1}{2}, \theta + \frac{1}{2}] \) में एक एकसमान बेटान से प्रेषण है, तो \(-\infty < \theta < \infty\) सम्मिलित एक अवलोकन प्राप्त है।

1. प्रतिविद्या \( \theta \) का अनन्त आक्लाद है।
2. प्रतिविद्या \( \theta \) का अनन्त आक्लाद है।
3. प्रतिविद्या \( \theta \) का एकसमानत: न्यूनतम प्रसरण अनन्त आक्लाद नहीं है।
4. प्रतिविद्या \( \theta \) का एकसमानत: न्यूनतम प्रसरण अनन्त आक्लाद नहीं है।

110. Let \( X_1, X_2, \ldots, X_{25} \) be i.i.d. observations from a uniform distribution on the interval \( [\theta - \frac{1}{2}, \theta + \frac{1}{2}] \) where \(-\infty < \theta < \infty\) is an unknown parameter. Then the

1. sample mean is an unbiased estimate for \( \theta \).
2. sample median is an unbiased estimate for \( \theta \).
3. sample mean is not the uniformly minimum variance unbiased estimate for \( \theta \).
4. sample median is not the uniformly minimum variance unbiased estimate for \( \theta \).

111. यदि \( X \) का घनांक \( f(x; \lambda) = \lambda e^{-\lambda x}, x > 0, \) तथा \( \lambda > 0 \) अवलोकन है। \( \text{जब } k < X \leq k + 1, k = 0, 1, 2, \ldots \). \( X \) की विभाजन करने से \( Y = k \) मिलता है। \( Y \) के अंश से \( Y_1, Y_2, \ldots, Y_n \) एक प्रारूपिक प्रतिविद्या प्राप्त होता है। \( \text{गानें कि } \bar{Y} = \frac{1}{n} \sum_{i=1}^{n} Y_i \) \( \text{तो } \lambda \) का आखून विधि आक्लाद \( \hat{\lambda} \) है।

1. \( \hat{\lambda} = \frac{1}{\bar{Y}} \)
2. \( \hat{\lambda} = \frac{1}{\bar{Y}} + 1 \)
3. \( \hat{\lambda} = \ln \left( 1 + \frac{1}{\bar{Y}} \right) \)
4. अधिकतम संभावितिता आक्लाद के समान।

111. Suppose \( X \) has density \( f(x; \lambda) = \lambda e^{-\lambda x}, x > 0, \) where \( \lambda > 0 \) is unknown. \( X \) is discretized to give \( Y = k \) if \( k < X \leq k + 1, k = 0, 1, 2, \ldots \). A random sample \( Y_1, Y_2, \ldots, Y_n \) is available from the distribution of \( Y \). Let \( \bar{Y} = \frac{1}{n} \sum_{i=1}^{n} Y_i \). Then the method of moments estimator \( \hat{\lambda} \) of \( \lambda \) is
12. Let $X_1, X_2, \ldots, X_n$ be i.i.d. observations from a distribution with continuous probability density function $f$ which is symmetric around 0 i.e., $f(x - \theta) = f(\theta - x)$ for all real $x$.

Consider the test $H_0: \theta = 0$ vs $H_A: \theta > 0$ and the sign test statistic $S_n = \sum_{i=1}^{n} \text{sign}(X_i)$ where

$$\text{sign}(x) = \begin{cases} 1, & \text{if } x > 0 \\ 0, & \text{if } x = 0 \\ -1, & \text{if } x < 0 \end{cases}$$

Let $z_{1-\alpha}$ be the upper $100(1 - \alpha)$th percentile of the standard normal distribution where $0 < \alpha < 1$. Which of the following is/are correct?

1. If $\theta = 0$, then $\lim_{n \to \infty} P\left(S_n > \sqrt{n}z_{1-\alpha}\right) = 1$.
2. If $\theta = 0$, then $\lim_{n \to \infty} P\left(S_n > \sqrt{n}z_{1-\alpha}\right) = \alpha$. 

12. Let $X_1, X_2, \ldots, X_n$ be observations from a distribution with continuous probability density function $f$ which is symmetric around 0 i.e., $f(x - \theta) = f(\theta - x)$ for all real $x$.

Consider the test $H_0: \theta = 0$ vs $H_A: \theta > 0$ and the sign test statistic $S_n = \sum_{i=1}^{n} \text{sign}(X_i)$ where

$$\text{sign}(x) = \begin{cases} 1, & \text{if } x > 0 \\ 0, & \text{if } x = 0 \\ -1, & \text{if } x < 0 \end{cases}$$

Let $z_{1-\alpha}$ be the upper $100(1 - \alpha)$th percentile of the standard normal distribution where $0 < \alpha < 1$. Which of the following is/are correct?

1. If $\theta = 0$, then $\lim_{n \to \infty} P\left(S_n > \sqrt{n}z_{1-\alpha}\right) = 1$.
2. If $\theta = 0$, then $\lim_{n \to \infty} P\left(S_n > \sqrt{n}z_{1-\alpha}\right) = \alpha$.
3. If $\theta > 0$, then $\lim_{n \to \infty} P\left\{ S_n > \sqrt{n} z_\theta \right\} = 1$.

4. If $\theta > 0$, then $\lim_{n \to \infty} P\left\{ S_n > \sqrt{n} z_\theta \right\} = \alpha$.

113. \( X_1, X_2, \ldots, X_{10} \sim N(\theta, \sigma^2) \), $\sigma^2 = 10$ be a random sample. Consider a half normal prior $f(\theta) = \frac{1}{\sqrt{2\pi \sigma^2}} e^{-\frac{\theta^2}{2\sigma^2}}$, $\sigma^2 = 10$. Let $\hat{\theta} = \bar{X} = \frac{1}{10} \sum_{i=1}^{10} X_i$. Then

1. $\hat{\theta} = \bar{X}$
2. $\hat{\theta} = \frac{20 \bar{X}}{21}$
3. $\hat{\theta} \leq \bar{X}$ if $\bar{X} \geq 0$
4. $\hat{\theta} \geq \bar{X}$ if $\bar{X} \leq 0$

113. Suppose $X_1, X_2, \ldots, X_{10}$ is a random sample from $N(\theta, \sigma^2)$, $\sigma^2 = 10$. Consider the prior for $\theta$, $\theta \sim N(0, \tau^2)$, $\tau^2 = 20$. Let $\bar{X} = \frac{1}{10} \sum_{i=1}^{10} X_i$. Then the mode $\hat{\theta}$ of the posterior distribution for $\theta$ satisfies:

1. $\hat{\theta} = \bar{X}$
2. $\hat{\theta} = \frac{20 \bar{X}}{21}$
3. $\hat{\theta} \leq \bar{X}$ if $\bar{X} \geq 0$
4. $\hat{\theta} \geq \bar{X}$ if $\bar{X} \leq 0$

114. \((X, Y)\) पर किये गये निम्न पाँच प्रश्नों पर विचार करें: \((0, 1), (1, 2), (2, 3), (3, 2), (4, 1)\)। सो

1. $X$ पर $Y$ का लघुगुणांक-वर्ग रैखिक समाधान है $Y = \frac{9}{5}$।
2. $Y$ पर $X$ का लघुगुणांक-वर्ग रैखिक समाधान है $X = 2$
3. $X$ व $Y$ के बीच का सहसंबंध गुणांक 0 है।
4. $X$ व $Y$ के बीच का सहसंबंध गुणांक +1 है।

114. Consider the following five observations on \((X, Y)\): \((0, 1), (1, 2), (2, 3), (3, 2), (4, 1)\). Then

1. The least-square linear regression of $Y$ on $X$ is $Y = \frac{9}{5}$.
2. The least-square linear regression of $X$ on $Y$ is $X = 2$.
3. The correlation coefficient between $X$ and $Y$ is 0.
4. The correlation coefficient between $X$ and $Y$ is +1.
115. 

\[ Y_i = \mu + \epsilon_i, \quad Y_{i+1} - \mu = \rho (Y_i - \mu) + \sqrt{1-\rho^2} \epsilon_{i+1}, \quad i = 1, 2, ..., n-1 \]

\[ T = -\frac{1}{n} \sum_{i=1}^{n} Y_i \] \( \text{and} \quad 0 < \rho < 1 \quad \text{and} \quad \sigma^2 > 0 \quad \text{then} \quad n \geq 2 \text{ to suit} \]

1. \( T \) is a normal distribution.
2. \( T \) has mean \( \mu \) and variance \( \sigma^2/n \).
3. \( E(T) = \mu, \quad \text{var}(T) > \sigma^2/n. \)
4. \( T \sim N(\mu, \sigma^2) \) where \( \delta^2 > \sigma^2/n \).

115. Suppose \( \epsilon_1, \epsilon_2, ..., \epsilon_n \) are i.i.d. \( N(0, \sigma^2) \). Consider \( Y_1, Y_2, ..., Y_n \) defined by

\[ Y_i = \mu + \epsilon_i, \quad Y_{i+1} - \mu = \rho (Y_i - \mu) + \sqrt{1-\rho^2} \epsilon_{i+1}, \quad i = 1, 2, ..., n-1. \]

Let \( T = -\frac{1}{n} \sum_{i=1}^{n} Y_i \). Suppose \( 0 < \rho < 1 \) and \( \sigma^2 > 0 \). Then for \( n \geq 2 \)

1. \( T \) has a normal distribution.
2. \( T \) has mean \( \mu \) and variance \( \sigma^2/n \).
3. \( E(T) = \mu, \quad \text{var}(T) > \sigma^2/n. \)
4. \( T \sim N(\mu, \sigma^2) \) where \( \delta^2 > \sigma^2/n \).

116. एक चुनाव में एक राजनीतिक पार्टी को लिए बहुमत की अनुपात \( p \) की आकलन करने के लिए उपलब्ध \( x \) नियन्त्रित वोटों की संख्या (SRSWOR) रखने की संभावना है, हर वोट का समान न्याय है।

साधारण अवधारणा: साधारण वोट का समान न्याय है।

साधारण अवधारणा: साधारण वोट का समान न्याय है।

साधारण अवधारणा: साधारण वोट का समान न्याय है।

साधारण अवधारणा: साधारण वोट का समान न्याय है।

1. \( p_1 \) एक अनुमानित आकलन है, \( p_2 \) नहीं।
2. \( \frac{p_1 \times p_2}{200} \) दोनों अनुमानित आकलन है।
3. \( p_1 \) और \( p_2 \) दोनों अनुमानित आकलन है, \( p_2 \) का प्रसरण \( p_1 \) को हुल्ला में बना लो तो क्या समान है।
4. वर्तनी एक का मत देने वाले \( x \) र दूसरी \( y \) मतदाताओं की अनुपात समान होने पर ही \( p_1 \) और \( p_2 \) के प्रसरण समान होगा।

116. In a survey to estimate the proportion \( p \) of votes that a party will poll in an election, statisticians A and B follow different sampling strategies as follows:

**Statistician A:** Selects a simple random sample without replacement (SRSWOR) of 200 voters, finds that \( x \) of them will vote for the party and estimates \( p \) by
Statistician B: Divides the voters’ list into Male and Female lists, selects 100 from each list by SRSWOR, finds that \( x_1, x_2 \) respectively will vote for the party and estimates \( p \) by

\[
p = \frac{x_1 + x_2}{200}.
\]

The number of voters in the two lists are the same. Then

1. \( p_1 \) is an unbiased estimate but \( p_2 \) is not.
2. \( p_1 \) and \( p_2 \) are both unbiased estimates.
3. \( p_1 \) and \( p_2 \) are both unbiased estimates, but \( p_2 \) has a smaller variance than \( p_1 \), or the same variance as \( p_1 \).
4. Variances of \( p_1, p_2 \) are the same only if the proportions of male and female voters who vote for the party are the same.

117. 1, 2, ..., 5 से आक्षेप 5 उपयोग पुस्तक, दो खण्ड गाले निन्हे खण्ड अभिव्यक्तिप्रदा पर विचारे:

खण्ड I: \{1, 2, 3\}; खण्ड II: \{1, 4, 5\}

किन्हे में कोन सा/से कहने सही है/है?

1. अभिव्यक्तिप्रदा संबंध है।
2. यहीं \( \sigma^2 \) एक प्रश्न का प्रस्तर है, एक प्रारंभिक उपयोग विश्लेषण के क्षेत्र में रैखिक अभिव्यक्ति आकलन का प्रस्तर या तो \( 2\sigma^2 \) या \( 4\sigma^2 \) है।
3. अभिव्यक्तिप्रदा, जिसकी प्रश्नात्मक रूप से सवैधानिक है, द्वारा एकलह ऐसा कोई अद्वितीय रैखिक पतल नहीं है।
4. पुस्तक के साथ सहचरी लक्ष्य प्रति पुस्तक अभिव्यक्ति है।

117. Consider the following block design involving 5 treatments, labelled 1, 2, ..., 5, and two blocks:
Block I: \{1, 2, 3\}; Block II: \{1, 4, 5\}.
Which of the following statements is/are true?

1. The design is connected.
2. The variance of the best linear unbiased estimator of an elementary treatment contrast is either \( 2\sigma^2 \) or \( 4\sigma^2 \), where \( \sigma^2 \) is the variance of an observation.
3. There is no non-trivial linear function of observations collected through the design whose expectation is identically equal to zero.
4. The degrees of freedom associated with the error is zero.

118. मानना कि हमारे पास एक आंकड़ा समुच्छय है जिसमें 25 प्रश्न हैं, जिसका हर एक का मूल्य 0 या 1 है।

1. आंकड़ों का माध्य उनके प्रश्न के अधिक नहीं हो सकता।
2. आंकड़ों का माध्य उनके प्रश्न के कम नहीं हो सकता।
3. जब माध्य एवं प्रश्न समान हैं, इसका मतलब है कि माध्य मूल्य है।
4. प्रश्न शून्य तभी होगा जब माध्य 0 या 1 होगा।
118. Suppose that we have a data set consisting of 25 observations, where each value is either 0 or 1.

1. The mean of the data cannot be larger than the variance.
2. The mean of the data cannot be smaller than the variance.
3. The mean being same as the variance implies that the mean is zero.
4. The variance will be 0 if and only if the mean is either 1 or 0.

119. यद्यपयः \( x_1 + x_2 \geq 5 \), \( 4x_1 - x_2 \leq 15 \) एवं \( 4x_2 - x_1 \leq 15 \) को पूरा करने वाले \( x_1 \geq 0 \) एवं \( x_2 \geq 0 \) पर लिखित \( 3x_1 + 2x_2 \) का अधिकतम मान है 25।

1. \( 3x_1 + 2x_2 \) का अधिकतम मान है 25।
2. \( 3x_1 + 2x_2 \) का न्यूनतम मान है 11।
3. \( 3x_1 + 2x_2 \) का कोई न्यूनतम अधिकतम नहीं है।
4. \( 3x_1 + 2x_2 \) का कोई न्यूनतम अधिकतम नहीं है।

119. Consider the variables \( x_1 \geq 0 \) and \( x_2 \geq 0 \) satisfying the constraints \( x_1 + x_2 \geq 5 \), \( 4x_1 - x_2 \leq 15 \) and \( 4x_2 - x_1 \leq 15 \). Which of the following statements is/are correct?

1. The maximum value of \( 3x_1 + 2x_2 \) is 25
2. The minimum value of \( 3x_1 + 2x_2 \) is 11
3. \( 3x_1 + 2x_2 \) has no finite maximum
4. \( 3x_1 + 2x_2 \) has no finite minimum

120. एक लेखक दर्शाते एक प्राणी के मात्र 20% बढ़ते हैं 12 मिनट में एक खालित, लेकिन उनकी सभी प्राणी गाति, प्रति 8 मिनट में एक सेम लेने की जाती है। यदि आमतौर पर 20% बढ़ती है तो, तथ्य के अनुसार \( 1. \) प्राणी \( 2. \) प्राणी \( 3. \) प्राणी \( 4. \) प्राणी

1. \( \text{प्राणी} \text{ में कृति की माढ़ संख्या में बढ़ता 2 है।} \)
2. \( \text{प्राणी} \text{ में ग्राहक की माढ़ संख्या में बढ़ता 4 है।} \)
3. \( \text{प्राणी} \text{ में ग्राहक तक विशेष ग्रहण ग्राहक माढ़ में बढ़ता 16 मिनट है।} \)
4. \( \text{प्राणी} \text{ में ग्राहक तक विशेष ग्रहण ग्राहक माढ़ में बढ़ता 24 मिनट है।} \)

120. In a system with a single server, suppose that customers arrive at a Poisson rate of 1 person every 12 minutes and are serviced at the Poisson rate of 1 service every 8 minutes. If the arrival rate increases by 20% then in the steady state

1. the increase in the average number of customers in the system is 2,
2. the increase in the average number of customers in the system is 4,
3. the increase in the average time spent by a customer in the system is 16 minutes.
4. the increase in the average time spent by a customer in the system is 24 minutes.
This Test Booklet will contain 120 (20 Part ‘A’+40 Part ‘B+60 Part ‘C’) Multiple Choice Questions (MCQs) Both in Hindi and English. Candidates are required to answer 15 in part ‘A’, 25 in Part ‘B’ and 20 questions in Part ‘C’ respectively (No. of questions to attempt may vary from exam to exam). In case any candidate answers more than 15, 25 and 20 questions in Part A, B and C respectively only first 15, 25 and 20 questions in Parts A, B and C respectively will be evaluated. Each questions in Parts ‘A’ carries two marks, Part ‘B’ three marks and Part ‘C’ 4.75 marks respectively. There will be negative marking @0.5 marks in Part ‘A’ and 0.75 in part ‘B’ for each wrong answers. Below each question in Part ‘A’ and Part ‘B’, four alternatives or responses are given. Only one of these alternatives is the ‘CORRECT’ answer to the question. Part ‘C’ shall have one or more correct options. Credit in a question shall be given only on identification of ALL the correct options in Part ‘C’. No credit shall be allowed in a question if any incorrect option is marked as correct answer. No partial credit is allowed.

MODEL QUESTION PAPER

PART A

May be viewed under heading “General Science”

PART B

21. The sequence \( a_n = \frac{1}{n^2} + \frac{1}{(n+1)^2} + \ldots + \frac{1}{(2n)^2} \)

1. converges to 0
2. converges to 1/2
3. converges to 1/4
4. does not converge.

22. Let \( x_n = n^{1/n} \) and \( y_n = (n!)^{1/n} \), \( n \geq 1 \) be two sequences of real numbers. Then

1. \((x_n)\) converges, but \((y_n)\) does not converge
2. \((y_n)\) converges, but \((x_n)\) does not converge
3. Both \((x_n)\) and \((y_n)\) converge
4. Neither \((x_n)\) nor \((y_n)\) converges

23. The set \( \{x \in \mathbb{R} : x \sin x \leq 1, x \cos x \leq 1 \} \subset \mathbb{R} \) is
1. A bounded closed set
2. A bounded open set
3. An unbounded closed set.
4. An unbounded open set.

24. Let \(f : [0,1] \to \mathbb{R} \) be continuous such that \(f(t) \geq 0\) for all \(t\) in \([0, 1]\). Define

\[
g(x) = \int_0^x f(t) \, dt \quad \text{then}
\]

1. \(g\) is monotone and bounded
2. \(g\) is monotone, but not bounded
3. \(g\) is bounded, but not monotone
4. \(g\) is neither monotone nor bounded

25. Let \(f\) be a continuous function on \([0, 1]\) with \(f(0) = 1\). Let \(G(a) = \frac{1}{a} \int_0^a f(x) \, dx\)

1. \(\lim_{a \to 0} G(a) = \frac{1}{2}\)
2. \(\lim_{a \to 0} G(a) = 1\)
3. \(\lim_{a \to 0} G(a) = 0\)
4. The limit \(\lim_{a \to 0} G(a)\) does not exist

26. Let \(\alpha_n = \sin \left( \frac{1}{n^2} \right), n = 1, 2, \ldots\). Then

1. \(\sum_{n=1}^{\infty} \alpha_n\) converges
2. \(\limsup_{n \to \infty} \alpha_n = \liminf_{n \to \infty} \alpha_n\)
3. \(\lim_{n \to \infty} \alpha_n = 1\)
4. \(\sum_{n=1}^{\infty} \alpha_n\) diverges
If, for $x \in \mathbb{R}$, $\varphi(x)$ denotes the integer closest to $x$ (if there are two such integers take the larger one), then $\int_{10}^{12} \varphi(x) \, dx$ equals

$$
\begin{align*}
1 & \quad 22 \\
2 & \quad 11 \\
3 & \quad 20 \\
4 & \quad 12
\end{align*}
$$

Let $P$ be a polynomial of degree $k > 0$ with a non-zero constant term. Let $f_n(x) = P(\frac{x}{n}) \quad \forall x \in (0, \infty)$

\[1 \lim_{n \to \infty} f_n(x) = \infty \quad \forall x \in (0, \infty)\]

\[2 \exists x \in (0, \infty) \text{ such that } \lim_{n \to \infty} f_n(x) > P(0)\]

\[3 \lim_{n \to \infty} f_n(x) = 0 \quad \forall x \in (0, \infty)\]

\[4 \lim_{n \to \infty} f_n(x) = P(0) \quad \forall x \in (0, \infty)\]

Let $C[0, 1]$ denote the space of all continuous functions with supremum norm.

Then, $K = \{ f \in [0, 1] : \lim_{n \to \infty} f^{x+y+z} = \hat{u} \}$ is a vector space but not closed in $C[0,1]$.

1. closed but does not form a vector space.
2. a closed vector space but not an algebra.
3. a closed algebra.

Let $u, v, w$ be three points in $\mathbb{R}^3$ not lying in any plane containing the origin.

Then

\[1 \quad \alpha_1 u + \alpha_2 v + \alpha_3 w = 0 \Rightarrow \alpha_1 = \alpha_2 = \alpha_3 = 0\]

\[2 \quad u, v, w \text{ are mutually orthogonal}\]

\[3 \quad \text{one of } u, v, w \text{ has to be zero}\]

\[4 \quad u, v, w \text{ cannot be pairwise orthogonal}\]
31. Let \( x, y \) be linearly independent vectors in \( \mathbb{R}^2 \) suppose \( T: \mathbb{R}^2 \to \mathbb{R}^2 \) is a linear transformation such that \( Ty = \alpha x \) and \( Tx = 0 \) Then with respect to some basis in \( \mathbb{R}^2 \), \( T \) is of the form

1. \( \begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix} \), \( a > 0 \)
2. \( \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} \), \( a, b > 0; a \neq b \)
3. \( \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \)
4. \( \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \)

32. Suppose \( A \) is an \( n \times n \) real symmetric matrix with eigenvalues \( \lambda_1, \lambda_2, \ldots, \lambda_n \) then

1. \( \prod_{i=1}^{n} \lambda_i < \det(A) \)
2. \( \prod_{i=1}^{n} \lambda_i > \det(A) \)
3. \( \prod_{i=1}^{n} \lambda_i = \det(A) \)
4. if \( \det(A) = 1 \) then \( \lambda_j = 1 \) for \( j = 1, \ldots, n \).

33. Let \( f \) be analytic on \( D = \{ z \in \mathbb{C} : |z| < 1 \} \) and \( f(0) = 0 \).

Define

\[
g(z) = \begin{cases} \frac{f(z)}{z}; & z \neq 0 \\ f'(0); & z = 0 \end{cases}
\]

Then

1. \( g \) is discontinuous at \( z = 0 \) for all \( f \)
2. \( g \) is continuous, but not analytic at \( z = 0 \) for all \( f \)
3. \( g \) is analytic at \( z = 0 \) for all \( f \)
4. \( g \) is analytic at \( z = 0 \) only if \( f'(0) = 0 \)
34. Let \( \Omega \subseteq \mathbb{C} \) be a domain and let \( f(z) \) be an analytic function on \( \Omega \) such that 

\[
|f(z)| = |\sin z| \quad \text{for all } z \in \Omega
\]

then

1. \( f(z) = \sin z \) for all \( z \in \Omega \)
2. \( f(z) = \sin \left( \overline{z} \right) \) for all \( z \in \Omega \).
3. there is a constant \( c \in \mathbb{C} \) with \( |c| = 1 \) such that \( f(z) = c \sin z \) for all \( z \in \Omega \)
4. such a function \( f(z) \) does not exist

35. The radius of convergence of the power series

\[
\sum_{n=0}^{\infty} (4n^4 - n^3 + 3) z^n
\]

is

1. 0
2. 1
3. 5
4. \( \infty \)

36. Let \( \mathbb{F} \) be a finite field such that for every \( a \in \mathbb{F} \) the equation \( x^2 = a \) has a solution in \( \mathbb{F} \). Then

1. the characteristic of \( \mathbb{F} \) must be 2
2. \( \mathbb{F} \) must have a square number of elements
3. the order of \( \mathbb{F} \) is a power of 3
4. \( \mathbb{F} \) must be a field with prime number of elements

37. Let \( \mathbb{F} \) be a field with \( 5^{12} \) elements. What is the total number of proper subfields of \( \mathbb{F} \)?

1. 3
2. 6
3. 8
4. 5
38. Let \( K \) be an extension of the field \( \mathbb{Q} \) of rational numbers
1. If \( K \) is a finite extension then it is an algebraic extension
2. If \( K \) is an algebraic extension then it must be a finite extension
3. If \( K \) is an algebraic extension then it must be an infinite extension
4. If \( K \) is a finite extension then it need not be an algebraic extension

39. Consider the group \( S_9 \) of all the permutations on a set with 9 elements. What is the largest order of a permutation in \( S_9 \)?
1. 21
2. 20
3. 30
4. 14

40. Suppose \( V \) is a real vector space of dimension 3. Then the number of pairs of linearly independent vectors in \( V \) is
1. one
2. infinity
3. \( e^3 \)
4. 3

41. Consider the differential equation
\[
\frac{dy}{dx} = y^2, \ (x, y) \in \mathbb{R} \times \mathbb{R}.
\]
Then,
1. all solutions of the differential equation are defined on \((-\infty, \infty)\).
2. no solution of the differential equation is defined on \((-\infty, \infty)\).
3. the solution of the differential equation satisfying the initial condition \( y(x_0) = y_0, \ y_0 > 0 \), is defined on \((-\infty, x_0 + \frac{1}{y_0})\).
4. the solution of the differential equation satisfying the initial condition \( y(x_0) = y_0, \ y_0 > 0 \), is defined on \( x_0 - \frac{1}{y_0}, \frac{\delta}{\delta} \).

42. The second order partial differential equation
\[
(1 - \sqrt{xy}) \frac{\partial^2 u}{\partial x^2} + 2 \frac{\partial^2 u}{\partial x \partial y} + (1 + \sqrt{xy}) \frac{\partial^2 u}{\partial y^2} = 0
\]
is
1. hyperbolic in the second and the fourth quadrants
2. elliptic in the first and the third quadrants
3. hyperbolic in the second and elliptic in the fourth quadrant
4. hyperbolic in the first and the third quadrants

43. A general solution of the equation \( \frac{\partial u(x, y)}{\partial x} + u(x, y) = e^{-x} \) is

1. \( u(x, y) = e^{-x}f(y) \)
2. \( u(x, y) = e^{-x}f(y) + xe^x \)
3. \( u(x, y) = e^{x}f(y) + xe^{-x} \)
4. \( u(x, y) = e^{x}f(y) + xe^{-x} \)

44. Consider the application of Trapezoidal and Simpson’s rules to the following integral
\[ \int_{0}^{a} (2x^3 - 3x^2 + 5x + 1) \, dx \]

1. Both Trapezoidal and Simpson’s rules will give results with same accuracy.
2. The Simpson’s rule will give more accuracy than the Trapezoidal rule but less accurate than the exact result.
3. The Simpson’s rule will give the exact result.
4. Both Trapezoidal rule and Simpson’s rule will give the exact results.

45. The integral equation
\[ g(x)y(x) = f(x) + \int_{\alpha}^{\beta} k(x,t)y(t) \, dt \]
with \( f(x) \), \( g(x) \) and \( k(x,t) \) as known functions, \( \alpha \) and \( \beta \) as known constants, and \( \lambda \) as a known parameter, is a

1. linear integral equation of Volterra type
2. linear integral equation of Fredholm type
3. nonlinear integral equation of Volterra type
4. nonlinear integral equation of Fredholm type

46. Let \( y(x) = f(x) + \lambda \int_{a}^{b} k(x,t)y(t) \, dt \), where \( f(x) \) and \( k(x,t) \) are known functions, \( a \) and \( b \) are known constants and \( \lambda \) is a known parameter. If \( \lambda_i \) be the eigenvalues of the corresponding homogeneous equation, then the above integral equation has in general,

1. many solutions for \( \lambda \neq \lambda_i \)
2. no solution for \( \lambda \neq \lambda_i \)
3. a unique solution for \( \lambda = \lambda_i \)
4. either many solutions or no solution at all for \( \lambda = \lambda_i \), depending on the form of \( f(x) \)

47. The equation of motion of a particle in the x-z plane is given by
\[ \frac{d\vec{v}}{dt} = -\vec{v} - \hat{k} \]
with $\vec{v} = \alpha \hat{k}$, where $\alpha = \alpha(t)$ and $\hat{k}$ is the unit vector along the z-direction. If initially (i.e., $t = 0$) $\alpha = 1$, then the magnitude of velocity at $t = 1$ is

1. $2/e$
2. $(2+e)/3$
3. $(e-2)/e$
4. 1

48. Consider the functional

$$F(u,v) = \frac{\pi^2}{2} \int_0^{\pi/2} \left[ \left( \frac{du}{dx} \right)^2 + \left( \frac{dv}{dx} \right)^2 + 2u(x)v(x) \right] dx$$

with

$u(0) = 1, v(0) = -1$ and

$u\left( \frac{\pi}{2} \right) = 0, v\left( \frac{\pi}{2} \right) = 0$.

Then, the extremals satisfy

1. $u(\pi) = 1, v(\pi) = -1$
2. $u(\pi) + v(\pi) = 0, u(\pi) - v(\pi) = 2$
3. $u(p) = -1, v(p) = 1$
4. $u(\pi) + v(\pi) = -2, u(\pi) - v(\pi) = 0$

49. The pairs of observations on two random variables X and Y are

<table>
<thead>
<tr>
<th>X</th>
<th>2</th>
<th>5</th>
<th>7</th>
<th>11</th>
<th>13</th>
<th>19</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y</td>
<td>0</td>
<td>15</td>
<td>25</td>
<td>45</td>
<td>55</td>
<td>85</td>
</tr>
</tbody>
</table>

Then the correlation coefficient between X and Y is

1. 0
2. 1/5
3. 1/2
4. 1

50. Let $X_1$, $X_2$, $X_3$ be independent random variables with $P(X_i = +1) = P(X_i = -1) = 1/2$. Let $Y_1 = X_2X_3$, $Y_2 = X_1X_3$ and $Y_3 = X_1X_2$.

Then which of the following is NOT true?

1. $Y_i$ and $X_i$ have same distribution for $i = 1, 2, 3$
2. $(Y_1, Y_2, Y_3)$ are mutually independent
3. $X_1$ and $(Y_2, Y_3)$ are independent
4. $(X_1, X_2)$ and $(Y_1, Y_2)$ have the same distribution
51. Let $X$ be an exponential random variable with parameter $\lambda$. Let $Y = \lfloor X \rfloor$ where $\lfloor x \rfloor$ denotes the largest integer smaller than $x$. Then

1. $Y$ has a Geometric distribution with parameter $\lambda$.
2. $Y$ has a Geometric distribution with parameter $1 - e^{-\lambda}$.
3. $Y$ has a Poisson distribution with parameter $\lambda$.
4. $Y$ has mean $\lfloor 1/\lambda \rfloor$.

52. Consider a finite state space Markov chain with transition probability matrix $P=\left(\begin{matrix} p_{ij} \end{matrix}\right)$. Suppose $p_{ii} = 0$ for all states $i$. Then the Markov chain is

1. always irreducible with period 1.
2. may be reducible and may have period $> 1$.
3. may be reducible but period is always 1.
4. always irreducible but may have period $> 1$.

53. Let $X_1, X_2, \ldots, X_n$ be i.i.d. Normal random variables with mean 1 and variance 1. and let $Z_n = (X_1^2 + X_2 + \ldots + X_n)/n$. Then

1. $Z_n$ converges in probability to 1.
2. $Z_n$ converges in probability to 2.
3. $Z_n$ converges in distribution to standard normal distribution.
4. $Z_n$ converges in probability to Chi-square distribution.

54. Let $X_1, X_2, \ldots, X_n$ be a random sample of size $n (\geq 4)$ from uniform $(0,0)$ distribution. Which of the following is NOT an ancillary statistic?

1. $\frac{X_{(n)}}{X_{(1)}}$
2. $\frac{X_n}{X_1}$
3. $\frac{X_4 - X_1}{X_3 - X_2}$
4. $X_{(n)} - X_{(1)}$
Suppose \( X_1, X_2, \ldots, X_n \) are i.i.d, Uniform \( (0, q) \), \( \theta \in \{1, 2, \ldots\} \).

Then the MLE of \( \theta \) is
1. \( X_{(n)} \)
2. \( \bar{X} \)
3. \( \lfloor X_{(n)} \rfloor \) where \( [a] \) is the integer part of \( a \).
4. \( \lfloor X_{(n)} + 1 \rfloor \) where \( [a] \) is the integer part of \( a \).

Let \( X_1, X_2, \ldots, X_n \) be independent and identically distributed random variables with common continuous distribution function \( F(x) \). Let \( R_i = \text{Rank}(X_i), i = 1, 2, \ldots, n \). Then \( P(|R_n - R_1| \geq n - 1) \) is
1. \( 0 \)
2. \( \frac{1}{n(n-1)} \)
3. \( \frac{2}{n(n-1)} \)
4. \( \frac{1}{n} \)

A simple random sample of size \( n \) is drawn without replacement from a population of size \( N > n \). If \( \pi_i, i = 1, 2, \ldots, N \) and \( p_{ij}, i \neq j, i, j = 1, 2, \ldots, N \) denote respectively, the first and second order inclusion probabilities, then which of the following statements is NOT true?
1. \( \hat{\pi} \pi_i = n \)
2. \( \hat{\pi}_{i,j} = (n - 1)p_i \)
3. \( p_{i,j} \leq p_{i,j} \) for each pair \( (i, j) \)
4. \( p_{i,j} < p_i \) for each pair \( (i, j) \).

Consider a balanced incomplete block design with usual parameters \( v, b, r, k \) \( (\geq 2), \lambda \). Let \( t_i \) be the effect of the \( i^{th} \) treatment \( (i = 1, 2, \ldots, v) \) and \( \sigma^2 \) denote the variance of an observation. Then the variance of the best linear
unbiased estimator of \( \sum_{i=1}^{y} p_{i} l_{i} \), where \( \sum_{i=1}^{y} p_{i} = 0 \) and \( \sum_{i=1}^{y} p_{i}^{2} = 1 \), under the intra-block model, is

1. \( \left( \frac{\lambda v}{k} \right) \sigma^2 \)
2. \( 2\sigma^2 / r \)
3. \( \left( \frac{k}{\lambda v} \right) \sigma^2 \)
4. \( \left( \frac{2k}{\lambda v} \right) \sigma^2 \)

59. An aircraft has four engines – two on the left side and two on the right side. The aircraft functions only if at least one engine on each side functions. If the failures of engines are independent, and the probability of any engine failing in equal to \( p \), then the reliability of the aircraft is equal to

1. \( p^2 (1 - p^2) \)
2. \( 4C_2 p^3 (1 - p) \)
3. \( (1 - p^2)^2 \)
4. \( 1 - (1 - p^2)^2 \)

60. A company maintains EOQ model for one of its critical components. The setup cost is \( k \), unit production cost is \( c \), demand is \( a \) units per unit time, and \( h \) is the cost of holding one unit per unit time. In view of the criticality of the component the company maintains a safety stock of \( s \) units at all times. The economic order quantity for this problem is given by.

1. \( \sqrt{\frac{2ak}{h}} + s \)
2. \( s + \sqrt{\frac{2ak}{h}} \)
3. \( \sqrt{\frac{2ak}{h}} \)
4. \( \sqrt{\frac{2ak + s}{h}} \)
PART C

61. Suppose \( \{a_n\}, \{b_n\} \) are convergent sequences of real numbers such that \( a_n > 0 \) and \( b_n > 0 \) for all \( n \).

Suppose \( \lim_{n \to \infty} a_n = a \) and \( \lim_{n \to \infty} b_n = b \). Let \( c_n = a_n/b_n \). Then

1. \( \{c_n\} \) converges if \( b > 0 \)
2. \( \{c_n\} \) converges only if \( a = 0 \)
3. \( \{c_n\} \) converges only if \( b > 0 \)
4. \( \limsup_{n \to \infty} c_n = \infty \) if \( b = 0 \).

62. Consider the power series \( \sum_{n=0}^{\infty} a_n x^n \)

where \( a_0 = 0 \) and \( a_n = \sin(n!)/n! \) for \( n \geq 1 \). Let \( R \) be the radius of convergence of the power series. Then

1. \( R \geq 1 \)
2. \( R \geq 2\pi \)
3. \( R \leq 4\pi \)
4. \( R \geq \pi \).

63. Suppose \( f \) is an increasing real-valued function on \([0, \infty)\) with \( f(x) > 0 \ \forall x \) and let

\[
g(x) = \frac{1}{x} \int_{0}^{x} f(u) \, du; \quad 0 < x < \infty.
\]

Then which of the following are true:

1. \( g(x) \leq f(x) \) for all \( x \in (0, \infty) \)
2. \( x g(x) \leq f(x) \) for all \( x \in (0, \infty) \)
3. \( x g(x) \geq f(0) \) for all \( x \in (0, \infty) \)
4. \( y g(y) - x g(x) \leq (y-x)f(y) \) for all \( x < y \).

64. Let \( f : [0, 1] \to \mathbb{R} \) be defined by

\[
f(x) = \begin{cases} 
  x \cos(\pi/2x) & \text{if } x \neq 0, \\
  0 & \text{if } x = 0.
\end{cases}
\]

Then

1. \( f \) is continuous on \([0, 1]\)
2. \( f \) is of bounded variation on \([0, 1]\)
3. \( f \) is differentiable on the open interval \((0, 1)\) and its derivative \( f' \) is bounded on \((0,1)\)
4. \( f \) is Riemann integrable on \([0, 1]\).

65. For any positive integer \( n \), let \( f_n : [0, 1] \to \mathbb{R} \) be defined by
\[ f_n(x) = \frac{x}{nx+1} \text{ for } x \in [0,1]. \]

Then
1. the sequence \( \{f_n\} \) converges uniformly on \([0, 1]\)
2. the sequence \( \{ f'_n \} \) of derivatives of \( \{f_n\} \) converges uniformly on \([0, 1]\)
3. the sequence \( \left\{ \int_0^1 f_n(x) \, dx \right\} \) is convergent
4. the sequence \( \left\{ \int_0^1 f'_n(x) \, dx \right\} \) is convergent.

66. Let \( f: [0, \infty) \to \mathbb{R} \) and \( g: [0, \infty) \to \mathbb{R} \) be continuous functions satisfying
\[
\int_0^1 t^3 \, dt = x^3 (1 + x)^3 \quad \text{and} \quad \int_0^{x^2 (1+x)} g(t) \, dt = x \quad \text{for all } x \in [0, \infty].
\]

Then \( f(2) + g(2) \) is equal to
1. 0
2. 5
3. 6
4. 11.

67. Consider \( f: \mathbb{R}^2 \to \mathbb{R} \) defined by \( f(0, 0) = 0 \) and
\[ f(x, y) = \frac{x^2 y}{x^2 + y^2} \text{ for } (x, y) \neq (0, 0). \]

Then which of the following statements is correct?
1. Both the partial derivatives of \( f \) at \((0, 0)\) exist
2. The directional derivative \( D_u f(0, 0) \) of \( f \) at \((0, 0)\) exists for every unit vector \( u \)
3. \( f \) is continuous at \((0, 0)\)
4. \( f \) is differentiable at \((0, 0)\).

68. Let \( f: \mathbb{R}^2 \to \mathbb{R} \) and \( g: \mathbb{R}^2 \to \mathbb{R} \) be defined by
\[ f(x, y) = |x| + |y| \text{ and } g(x, y) = |xy|. \]

Then
1. \( f \) is differentiable at \((0, 0)\), but \( g \) is not differentiable at \((0, 0)\)
2. \( g \) is differentiable at \((0, 0)\), but \( f \) is not differentiable at \((0, 0)\)
3. Both \( f \) and \( g \) are differentiable at \((0, 0)\)
4. Both \( f \) and \( g \) are continuous at \((0, 0)\).

69. Decide for which of the functions \( F: \mathbb{R}^3 \to \mathbb{R}^3 \) given below, there exists a function \( f: \mathbb{R}^3 \to \mathbb{R} \) such that \( (\nabla f)(x) = F(x) \).
1. \((4xyz - z^2 - 3y^2, 2x^2z - 6xy + 1, 2x^2y - 2xz - 2)\)
2. \((x, xy, xyz)\)
3. \((1, 1, 1)\)
4. \((xyz, yz, z)\).

70. Let \(f: \mathbb{R}^n \to \mathbb{R}\) be the function defined by the rule \(f(x) = x.b\), where \(b \in \mathbb{R}^n\) and \(x.b\) denotes the usual inner product. Then

1. \([f'(x)](b) = b \cdot b\)
2. \([f'(x)](x) = \frac{x \cdot x}{2}, x \in \mathbb{R}^n\)
3. \([f'(0)](e_1) = b \cdot e_1\), where \(e_1 = (1, 0, \ldots, 0) \in \mathbb{R}^n\).
4. \([f'(e_1)](e_j) = 0, j \neq 1\), where \(e_j = (0, \ldots, 1, \ldots, 0)\) with 1 in the \(j^{th}\) slot.

71. Consider the subsets \(A\) and \(B\) of \(\mathbb{R}^2\) defined by

\[
A = \left\{ \left( x, x \sin \frac{1}{x} \right) : x \in (0, 1] \right\} \text{ and } B = A \cup \{(0, 0)\}.
\]

Then
1. \(A\) is compact
2. \(A\) is connected
3. \(B\) is compact
4. \(B\) is connected.

72. Let \(f = \mathbb{R} \to \mathbb{R}\) be a continuous function. Which of the following is always true?

1. \(f^{-1}(U)\) is open for all open sets \(U \subseteq \mathbb{R}\)
2. \(f^{-1}(C)\) is closed for all closed sets \(C \subseteq \mathbb{R}\)
3. \(f^{-1}(K)\) is compact for all compact sets \(K \subseteq \mathbb{R}\)
4. \(f^{-1}(G)\) is connected for all connected sets \(G \subseteq \mathbb{R}\).

73. Let \(A\) be an \(n \times n\) matrix, \(n \geq 2\), with characteristic polynomial \(x^{n-2}(x^2 - 1)\). Then

1. \(A^n = A^{n-2}\)
2. Rank of \(A\) is 2
3. Rank of \(A\) is at least 2
4. There exist nonzero vectors \(x\) and \(y\) such that \(A(x + y) = x - y\).

74. Let \(A, B\) and \(C\) be real \(n \times n\) matrices such that \(AB + B^2 = C\). Suppose \(C\) is nonsingular. Which of the following is always true?

1. \(A\) is nonsingular
2. \(B\) is nonsingular
3. \(A\) and \(B\) are both nonsingular
4. \(A + B\) is nonsingular.
75. Let V be a real vector space and let \{x_1, x_2, x_3\} be a basis for V. Then

1. \{x_1 + x_2, x_2, x_3\} is a basis for V
2. The dimension of V is 3
3. \(x_1, x_2, x_3\) are pairwise orthogonal
4. \{x_1 - x_2, x_2 - x_3, x_1 - x_3\} is a basis for V.

76. Consider the system of m linear equations in n unknowns given by \(Ax = b\), where \(A = (a_{ij})\) is a real \(m \times n\) matrix, \(x\) and \(b\) are \(n \times 1\) column vectors. Then

1. There is at least one solution
2. There is at least one solution if \(b\) is the zero vector
3. If \(m = n\) and if the rank of \(A\) is \(n\), then there is a unique solution
4. If \(m < n\) and if the rank of the augmented matrix \([A: b]\) equals the rank of \(A\), then there are infinitely many solutions.

77. Let V be the set of all real \(n \times n\) matrices \(A = (a_{ij})\) with the property that \(a_{ij} = -a_{ji}\) for all \(i, j = 1, 2, \ldots, n\). Then

1. V is a vector space of dimension \(n^2 - n\)
2. For every \(A\) in V, \(a_{ii} = 0\) for all \(i = 1, 2, \ldots, n\)
3. V consists of only diagonal matrices
4. V is a vector space of dimension \(\frac{n^2 - n}{2}\).

78. Let W be the set of all \(3 \times 3\) real matrices \(A = (a_{ij})\) with the property that \(a_{ij} = 0\) if \(i > j\) and \(a_{ii} = 1\) for all \(i\). Let \(B = (b_{ij})\) be a \(3 \times 3\) real matrix that satisfies \(AB = BA\) for all \(A\) in W. Then

1. Every \(A\) in W has an inverse which is in W.
2. \(b_{12} = 0\)
3. \(b_{13} = 0\)
4. \(b_{23} = 0\).

79. Let \(f(z)\) be an entire function with \(\text{Re}(f(z)) \geq 0\) for all \(z \in \mathbb{C}\). Then

1. \(\text{Im}(f(z)) \geq 0\) for all \(z \in \mathbb{C}\)
2. \(\text{Im}(f(z)) = \text{a constant}\)
3. \(f\) is a constant function
4. \(\text{Re}(f(z)) = |z|\) for all \(z \in \mathbb{C}\).

80. Let \(f\) be an analytic function defined on \(D = \{z \mid |z| < 1\}\) such that \(|f(z)| \leq 1\) for all \(z \in D\). Then

1. there exists \(z_0 \in D\) such that \(f(z_0) = 1\)
2. the image of \(f\) is an open set
3. \(f(0) = 0\)
4. \( f \) is necessarily a constant function.

81. Let \( f(z) = \frac{\sin z}{z^2} - \frac{\cos z}{z} \). Then

1. \( f \) has a pole of order 2 at \( z = 0 \)
2. \( f \) has a simple pole at \( z = 0 \)
3. \( \oint_{|z|=1} f(z)dz = 0 \), where the integral is taken anti-clockwise
4. the residue of \( f \) at \( z = 0 \) is \(-2\pi i\).

82. Let \( f \) be an analytic function defined on \( D = \{ z \in \mathbb{C}: |z| < 1 \} \). Then \( g : D \to \mathbb{C} \) is analytic if

1. \( g(z) = f(z) \) for all \( z \in D \)
2. \( g(z) = \overline{f(z)} \) for all \( z \in D \)
3. \( g(z) = \overline{f(z)} \) for all \( z \in D \)
4. \( g(z) = \overline{f(z)} \) for all \( z \in D \).

83. Which of the following statements involving Euler's function \( \phi \) is/are true?

1. \( \phi(n) \) is even as many times as it is odd
2. \( \phi(n) \) is odd for only two values of \( n \)
3. \( \phi(n) \) is even when \( n > 2 \)
4. \( \phi(n) \) is odd when \( n = 2 \) or \( n \) is odd.

84. Let \( p \) be a prime number and \( d \mid (p - 1) \). Then which of the following statements about the congruence \( x^d \equiv 1 \pmod{p} \) is/are true?

1. It does not have any solution
2. It has at most \( d \) incongruent solutions
3. It has exactly \( d \) incongruent solutions
4. It has at least \( d \) incongruent solutions.

85. Let \( K \) be a field, \( L \) a finite extension of \( K \) and \( M \) a finite extension of \( L \). Then

1. \([M:K] = [M:L] + [L:K]\)
2. \([M:K] = [M:L] \cdot [L:K]\)
3. \([M:L]\) divides \([M:K]\)
4. \([L:K]\) divides \([M:K]\).

86. Let \( R \) be a commutative ring and \( R[x] \) be the polynomial ring in one variable over \( R \).

1. If \( R \) is a U.F.D., then \( R[x] \) is a U.F.D.
2. If \( R \) is a P.I.D., then \( R[x] \) is a P.I.D.
3. If \( R \) is an Euclidean domain, then \( R[x] \) is an Euclidean domain
4. If \( R \) is a field, the \( R[x] \) is an Euclidean domain.
87. Let \( G \) be a group of order 56. Then

1. All 7-Sylow subgroups of \( G \) are normal
2. All 2-Sylow Subgroups of \( G \) are normal
3. Either a 7-Sylow subgroup or a 2-Sylow subgroup of \( G \) is normal
4. There is a proper normal subgroup of \( G \).

88. Which of the following statements is/are true?

1. 50! ends with an even number of zeros
2. 50! ends with a prime number of zeros
3. 50! ends with 10 zeros
4. 50! ends with 12 zeros.

89. Let \( X = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 = 1\} \)
\( Y = \{(x, y) \in \mathbb{R}^2 \mid |x| + |y| = 1\}, \) and
\( Z = \{(x, y) \in \mathbb{R}^2 \mid x^2 - y^2 = 1\}. \)

Then
1. \( X \) is not homeomorphic to \( Y \)
2. \( Y \) is not homeomorphic to \( Z \)
3. \( X \) is not homeomorphic to \( Z \)
4. No two of \( X, Y \) or \( Z \) are homeomorphic.

90. Let \( \tau_1, \tau_2 \) and \( \tau_3 \) be topologies on a set \( X \) such that \( \tau_1 \subseteq \tau_2 \subseteq \tau_3 \) and \( (X, \tau_2) \) is a compact Hausdorff space. Then

1. \( \tau_1 = \tau_2 \) if \( (X, \tau_1) \) is a Hausdorff space
2. \( \tau_1 = \tau_2 \) if \( (X, \tau_1) \) is a compact space
3. \( \tau_2 = \tau_3 \) if \( (X, \tau_3) \) is a Hausdorff space
4. \( \tau_2 = \tau_3 \) if \( (X, \tau_3) \) is a compact space.

91. The initial value problem \( \ddot{x}(t) = 3x^{2/3}, \ x(0) = 0; \)
in an interval around \( t = 0 \), has

1. no solution
2. a unique solution
3. finitely many linearly independent solutions
4. infinitely many linearly independent solutions.

92. For the system of ordinary differential equations:
\[
\frac{d}{dt} \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix},
\]

1. every solution is bounded
2. every solution is periodic
3. there exists a bounded solution
4. there exists a non periodic solution.

93. The kernel \( p(x, y) = \frac{y}{y^2 + x^2} \) is a solution of

1. the heat equation
2. the wave equation
3. the Laplace equation
4. the Lagrange equation.

94. The solution of the Laplace equation on the upper half plane, which takes the value \( \varphi(x) = e^x \) on the real line is

1. the real part of an analytic function
2. the imaginary part of an analytic function
3. the absolute value of an analytic function
4. an infinitely differentiable function.

95. Which of the following polynomials interpolate the data

<table>
<thead>
<tr>
<th>x</th>
<th>1</th>
<th>1/2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>3</td>
<td>-10</td>
<td>2</td>
</tr>
</tbody>
</table>

1. \( 3 + 26(x - 1) - \frac{53}{5}(x - 1)(x - 1) \)
2. \( 3(x - 1)(x - 1) - 10(x - 1)(x - 3) + 10(x - 3)(x - 1) \)
3. \( 3(x - \frac{1}{2})(x - 3) - 8(x - 1)(x - 3) + \frac{2}{5}(x - 1)(x - \frac{1}{2}) \)
4. \( (x - 3)(x + 10) + \frac{1}{2}(x + 10)(x - 2) + 3(x - 2)(x - 3) \).
96. The evaluation of the quantity $\sqrt{x+1} - 1$ near $x = 0$ is achieved with minimum loss of significant digits if we use the expression

1. $\sqrt{x+1} - 1$

2. $\frac{x}{\sqrt{x+1}+1}$

3. $\left(1 - \frac{1}{\sqrt{x+1}}\right) \sqrt{x+1}$

4. $\frac{x+2\sqrt{x+1}}{\sqrt{x+1}+1}$.

97. If $x(t)$ is an extremal of the functional $\int_{a}^{b} \left(\frac{1}{2} m(x)^2 - cx^2\right) dt$, where $a, b, c$ are arbitrary constants and $x = dx/dt$, then the function $x(t)$ satisfies

1. $m\ddot{x} + 2cx = 0$

2. $m\ddot{x} - 2cx = 0$

3. $m\dot{x}^2 + 2cx^2 = k_1$ with $k_1$ as an arbitrary constant

4. $x(t) = k_1 \sin\left(\frac{2c}{m} t + k_2\right)$ with $k_1$ and $k_2$ as arbitrary constants.

98. If $u(x)$ and $v(x)$ satisfying $u(0) = 1$, $v(0) = -1$, $u(\pi/2) = 0$ and $v((\pi/2)=0$ are the extremals of the functional $\int_{0}^{\pi/2} \left\{\left(\frac{du}{dx}\right)^2 + \left(\frac{dv}{dx}\right)^2 + 2uv\right\} dx$, then

1. $u(\frac{\pi}{4}) + v(\frac{\pi}{4}) = 0$

2. $u(\frac{\pi}{3}) - v(\frac{\pi}{3}) = 0$

3. $u(\frac{\pi}{4}) - v(\frac{\pi}{4}) = 1$

4. $u(\frac{\pi}{3}) + v(\frac{\pi}{3}) = 0$.

99. Consider the integral equation $y(x) = x^2 + \lambda \int_{0}^{1} xty(t)dt,$
where $\lambda$ is a real parameter. Then the Neumann series for the integral equation converges for all values of $\lambda$

1. except for $\lambda = 3$
2. lying in the interval $-3 < \lambda < 0$
3. lying in the interval $-3 < \lambda < 3$
4. lying in the interval $0 < \lambda < 3$.

100. The solution of the integral equation $\phi(x) = \frac{5x}{6} + \frac{1}{2} \int_0^1 xt\phi(t)dt$ satisfies

1. $\phi(0) + \phi(1) = 1$
2. $\phi\left(\frac{1}{2}\right) + \phi\left(\frac{1}{3}\right) = 1$
3. $\phi\left(\frac{1}{4}\right) + \phi\left(\frac{1}{2}\right) = 1$
4. $\phi\left(\frac{3}{4}\right) + \phi\left(\frac{1}{4}\right) = 1$.

101. A particle of unit mass is constrained to move on the plane curve $xy = 1$ under gravity $g$. Then

1. the kinetic energy of the system is $\frac{1}{2}(x^2 + y^2)$
2. the potential energy of the system is $\frac{g}{x}$
3. the Lagrangian of the particle is $\frac{1}{2}x^2(1 + x^{-4}) - \left(\frac{g}{x}\right)$
4. the Lagrangian of the particle is $\frac{1}{2}x^2(1 + x^{-4}) + \left(\frac{g}{x}\right)$.

102. Suppose a mechanical system has the single coordinate $q$ and Lagrangian $L = \frac{1}{4}q^2 - \frac{q^2}{9}$. Then

1. the Hamiltonian is $p^2 + \left(\frac{q^2}{9}\right)$
2. Hamilton’s equations are \( \dot{q} = 2p, \; \dot{p} = -(2/9)q \)

3. \( q \) satisfies \( \ddot{q} + (4/9)q = 0 \)

4. the path in the Hamiltonian phase-space, i.e. \( q - p \) plane is an ellipse.

103. Let \( X_1, \ldots, X_n \) be i.i.d. observations from a distribution with variance \( \sigma^2 \) \( (< \infty) \). Which of the following is/are unbiased estimator(s) of \( \sigma^2 \) ?

1. \( \frac{1}{n-1} \sum_{i=1}^{n} (X_i - \overline{X})^2 \)

2. \( \frac{1}{n} \sum_{i=1}^{n} (X_i - \overline{X})^2 \)

3. \( \left( \frac{n}{2} \right)^{-1} \frac{1}{2} \sum_{i=1}^{n} (X_i - X_j)^2 \)

4. \( \frac{1}{n} \sum_{i=1}^{n} X_i^2 - n\overline{X}^2 \).

104. Let \( X_1, X_2, \ldots \) be i.i.d. \( N(0,1) \) and let \( S_n = \sum_{i=1}^{n} X_i \) be the partial sums.

Which of the following is/are true?

1. \( \frac{S_n}{n} \rightarrow 0 \) almost surely

2. \( E\left( \frac{S_n}{n} \right) \rightarrow 0 \)

3. \( Var\left( \frac{S_n}{n} \right) \rightarrow 0 \)

4. \( Var\left( \frac{S_n^2}{n^2} \right) \rightarrow 0 \)

105. Let \( (X, Y) \) be a pair of independent random variables with \( X \) having exponential distribution with mean 1 and \( Y \) having uniform distribution on \( \{1, 2, \ldots, m\} \). Define \( Z = X + Y \). Then

1. \( E(Z|X) = X + \frac{m+1}{2} \)
2. \[ E(Z|Y) = 1 + \frac{m+1}{2} \]

3. \[ \text{Var}(Z|X) = \frac{m^2 - 1}{2} \]

4. \[ \text{Var}(Z|Y) = 2. \]

106. A simple symmetric random walk on the integer line is a Markov chain which is

1. recurrent
2. null recurrent
3. irreducible
4. positive recurrent.

107. Suppose \( X \) and \( Y \) are random variables with \( E(X) = E(Y) = 0, \text{Var}(X) = \text{Var}(Y) = 1 \) and \( \text{Cov}(X,Y) = 0.25 \). Then which of the following is/are always true?

1. \[ P \{ |X+2Y| \geq 4 \} \leq \frac{4}{16} \]
2. \[ P \{ |X+2Y| \geq 4 \} \leq \frac{5}{16} \]
3. \[ P \{ |X+2Y| \geq 4 \} \leq \frac{6}{16} \]
4. \[ P \{ |X+2Y| \geq 4 \} \leq \frac{7}{16} \]

108. Let \( X_1, \ldots, X_n \) be a random sample from uniform \((\theta, \theta + 1)\) distribution. Which of the following is/are maximum likelihood estimator(s) of \( \theta \)?

1. \( X_{(1)} \)
2. \( X_{(n)} \)
3. \( X_{(n)} - 1 \)
4. \( \frac{X_{(n)} + X_{(1)}}{2} - 0.5 \)

109. Let \( \overline{X} = (X_1, \ldots, X_n) \) be a random sample from uniform
(0. θ). Which of the following is/are uniformly most powerful size 
\(\alpha \left( 0 < \alpha < \frac{1}{2} \right)\) test(s) for testing \(H_0: \theta = \theta_0\) against \(H_1: \theta > \theta_0\)?

1. \(\phi_1(X) = 1\), if \(X_{(n)} > \theta_0\) or \(X_{(n)} < \theta_0\), \(\alpha \frac{1}{n}\)
   
   = 0, otherwise

2. \(\phi_2(X) = 1\), if \(X_{(n)} > \theta_0\)
   
   = \(\alpha\), if \(X_{(n)} \leq \theta_0\)

3. \(\phi_3(X) = 1\), if \(X_{(n)} > \theta_0 \alpha \frac{1}{n}\)
   
   = 0, if \(X_{(n)} \leq \theta_0 \alpha \frac{1}{n}\)

4. \(\phi_4(X) = 1\), if \(X_{(n)} < \theta_0 \left(\frac{\alpha}{2}\right) \frac{1}{n}\) or \(X_{(n)} > \theta_0 \left(1 - \frac{\alpha}{2}\right) \frac{1}{n}\)
   
   = 0, otherwise

110. Suppose \(X_{px1}\) has a \(N_p(\mu, I_p)\) distribution. The distribution of \(X^TAX\) is chi-square with \(r\) degrees of freedom only if

1. A is idempotent with rank \(r\)
2. Trace (A) = Rank (A) = \(r\)
3. A is positive definite
4. A is non-negative definite with rank \(r\).

111. Let \(X_1, X_2, \ldots, X_m\) be iid random variables with common continuous cdf \(F(x)\). Also let \(Y_1, Y_2, \ldots, Y_n\) be iid random variables with common continuous cdf \(G(x)\) and \(X's\) & \(Y's\) are independently distributed. For testing \(H_0: F(x) = G(x)\) for all \(x\) against \(H_1: F(x) \neq G(x)\) for at least one \(x\), which of the following test is/are used?

1. Wilcoxon signed rank test
2. Kolmogorov-Smirnov test
3. Wald-Wolfowitz run test
4. Sign test.

112. Random variables \(X\) and \(Y\) are such that \(E(X) = E(Y) = 0\), \(V(X) = V(Y) = 1\), correlation \((X, Y) = 0.5\). Then the

1. conditional distribution \(Y\) given \(X = x\) is normal with mean \(0.5x\) and variance \(0.75\)
2. least-squares linear regression of $Y$ on $X$ is $y=0.5x$ and of $X$ on $Y$ is $x=2y$

3. least-squares linear regression of $X$ on $Y$ is $x = 0.5y$ and of $Y$ on $X$ is $y = 2x$.

4. least-squares linear regression of $Y$ on $X$ is $y=0.5x$ and of $X$ on $Y$ is $x = 0.5y$.

113. X has a binomial $(5,p)$ distribution on which an observation $x=4$ has been made. In a Bayesian approach to the estimation of $p$, a beta $(2,3)$ prior distribution (with density proportional to $p(1-p)^2$) has been formulated. Then the posterior

1. distribution of $p$ is uniform on $(0.1)$

2. mean of $p$ is $\frac{6}{10}$

3. distribution of $p$ is beta $(6,4)$

4. distribution of $p$ is binomial $(10,0.5)$.

114. In a study of voter preferences in an election, the following data were obtained

<table>
<thead>
<tr>
<th>Gender</th>
<th>Party voting for</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>C</td>
<td>B</td>
</tr>
<tr>
<td>Male</td>
<td>250</td>
<td>250</td>
</tr>
<tr>
<td>Female</td>
<td>250</td>
<td>250</td>
</tr>
<tr>
<td>Total</td>
<td>500</td>
<td>500</td>
</tr>
</tbody>
</table>

Then the

1. chi-square statistic for testing no association between party and gender is 0.

2. expected frequency under the hypothesis of no association is 250 in each cell.

3. log-linear model for cell frequency $m_{ij}$, $\log(m_{ij}) = \text{constant}$, $i,j=1,2$, fits perfectly to the data.

4. chi-square test of no gender-party association with 1 degree of freedom has a p-value of 1.
115. Let \(X, Y\) and \(N\) be independent random variables with \(P(X=0) = \frac{1}{2} = 1 - P(X=1)\) and \(Y\) following Poisson with parameter \(\lambda > 0\) and \(N\) following normal with mean 0 and variance 1. Define
\[
Z = \begin{cases} 
Y & \text{if } X = 0 \\
N & \text{if } X = 1 
\end{cases}
\]
Then, the characteristic function of \(Z\) is given by
\[
\begin{align*}
1. & \quad \left(\frac{1}{2} + \frac{1}{2} e^{it}\right)e^{-\lambda(1-e^{it})}e^{-i^2/2} \\
2. & \quad e^{-\lambda(1-e^{it})}e^{-i^2/2} \\
3. & \quad \frac{e^{-\lambda(1-e^{it})} + e^{-i^2/2}}{2} \\
4. & \quad \left(\frac{1}{2} + \frac{1}{2} e^{it}\right)\left(\frac{e^{-\lambda(1-e^{it})} + e^{-i^2/2}}{2}\right) 
\end{align*}
\]

116. A simple random sample of size \(n\) is drawn from a finite population of \(N\) units, with replacement. The probability that the \(i^{th}\) (\(1 \leq i \leq N\)) unit is included in the sample is
\[
\begin{align*}
1. & \quad \frac{n}{N} \\
2. & \quad 1 - \left(1 - \frac{1}{N}\right)^n \\
3. & \quad \left(\frac{N-1}{N}\right)^n \\
4. & \quad \frac{n(n-1)}{N(N-1)} 
\end{align*}
\]

117. Under a balanced incomplete block design with usual parameters \(v, b, r, k, \lambda\), which of the following is/are true?
\[
\begin{align*}
1. & \quad \text{All treatment contrasts are estimable if } k \geq 2 \\
2. & \quad \text{The variance of the best linear unbiased estimator of any normalized treatment contrast is a constant depending only on the design parameters and the per observation variance} \\
3. & \quad \text{The covariance between the best linear unbiased estimators of two mutually orthogonal treatment contrasts is strictly positive} \\
4. & \quad \text{The variance of the best linear unbiased estimator of an elementary treatment contrast is strictly smaller than that under a randomized block design with replication } r. 
\end{align*}
\]
118. Consider a randomized (complete) block design with \( v > 2 \) treatments and \( r \geq 2 \) replicates. Which of the following statements is/are true?

1. The design is connected
2. The variance of the best linear unbiased estimator (BLUE) of every normalized treatment contrast is the same
3. The BLUE of any treatment contrast is uncorrelated with the BLUE of any contrast among replicate effects
4. The variance of the BLUE of any elementary treatment contrast is \( 2\sigma^2/r \), where \( \sigma^2 \) is the variance of an observation.

119. The starting and optimal tableaus of a minimization problem are given below. The variables are \( x_1, x_2 \) and \( x_3 \). The slack variables are \( S_1 \) and \( S_2 \).

### Starting Tableau

<table>
<thead>
<tr>
<th></th>
<th>( Z )</th>
<th>( x_1 )</th>
<th>( x_2 )</th>
<th>( x_3 )</th>
<th>( S_1 )</th>
<th>( S_2 )</th>
<th>RHS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Z</td>
<td>1</td>
<td>( a )</td>
<td>1</td>
<td>-3</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( S_1 )</td>
<td>0</td>
<td>( b )</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>( S_2 )</td>
<td>0</td>
<td>-1</td>
<td>2</td>
<td>-1</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

### Optimal Tableau

<table>
<thead>
<tr>
<th></th>
<th>( Z )</th>
<th>( x_1 )</th>
<th>( x_2 )</th>
<th>( x_3 )</th>
<th>( S_1 )</th>
<th>( S_2 )</th>
<th>RHS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Z</td>
<td>1</td>
<td>0</td>
<td>(-1/3)</td>
<td>(-11/3)</td>
<td>(-2/3)</td>
<td>0</td>
<td>(-4)</td>
</tr>
<tr>
<td>( x_1 )</td>
<td>0</td>
<td>( c )</td>
<td>( 2/3 )</td>
<td>( 2/3 )</td>
<td>( 1/3 )</td>
<td>0</td>
<td>( e )</td>
</tr>
<tr>
<td>( S_2 )</td>
<td>0</td>
<td>( d )</td>
<td>( 8/3 )</td>
<td>(-1/3)</td>
<td>( 1/3 )</td>
<td>1</td>
<td>3</td>
</tr>
</tbody>
</table>

Which of the following are the correct values of the unknowns \( a, b, c, d \) and \( e \)

1. \( a = 2, b = 3, c = 1, d = 0, e = 2 \)
2. \( a = 2, b = -3, c = 1, d = 0, e = -2 \)
3. \( a = -2, b = 3, c = 1, d = 0, e = 2 \)
4. \( a = -2, b = 3, c = -1, d = 0, e = 2 \)

120. Consider the following linear programming problem.

Minimize

\[
Z = x_1 + x_2
\]

subject to

\[
sx_1 + tx_2 \geq 1
\]

\[
x_1 \geq 0
\]

\( x_2 \) unrestricted.

The necessary and sufficient condition to make the LP

1. feasible is \( s \leq 0 \) and \( t = 0 \)
2. unbounded is \( s > t \) or \( t < 0 \)
3. have a unique solution is \( s = t \) and \( t > 0 \)
4. have a finite optimal solution is \( x_2 \geq 0 \).